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IDENTIFICATION OF BEHAVIORAL SIGNATURE: BENFORD'S LAW IN ONLINE AUCTIONS

Seokjoo Andrew Chang[1], Albert Chang[2]

[1] *Associate Professor, School of Business, State University of New York at Albany, USA*
[2] *Risk Analyst, Deloitte and Touche LLP*

Abstract

For a forensic implication of online auction market earnings, we study networked overlapping online auctions and underlying agent strategy. Bidder type identification provides efficient prior information for price formation process. Especially early-stage recognition of specific bidder types enables faster ex-ante revenue estimation. Characterizing behavioral pattern of bidding strategies, we identify unique digital signatures of heterogeneous bidder types. Given that the bidder types impose direct impact on the revenue, we further extend the conceptual domain to the potential bidding fraud which undermines overall revenue structure. We highlight the method of agent signature identification through Benford's law and power law. In our findings, there exists a bidder class which confirms the distributional pattern of Benford's law and their revenue impact is significant. Explicit characterization is conducted based on the power law. Participating agent strategy reflects their agent cost, surplus as well as market earnings.

Keywords

Benford's Law, Online Auctions, Institutional Bidding

1. Introduction

In today's e-business environment, the prominence of online auctions has created high degree of market liquidity for a broad range of goods and services. An interesting aspect of current e-auctions is that often multiple mechanisms exist concurrently or in overlapping manner. The coexistence of overlapping mechanisms can influence the behavioral pattern of participating agents and their strategy. Highlighting forensic implication of agent strategies, we study systematic ways to characterize and identify unique signatures of bidder behavior.

While our paper is motivated by a stream of concurrent auction mechanisms [2][10][11][15] and more recent view of competing sources [1][4][18][19], we shed further light on the forensic characterization and profiling of such signatures as well as early-stage identification of potentially fraudulent patterns [13]. Given the dynamic pricing nature of online auctions, participants' bidding strategy is a key component in price determination. Accurate identification and profiling of the strategies will help accounting buyer-side cost of bidding and resulting market earnings.

Applying k-means clustering algorithm, we characterize two heterogeneous segments of bidding strategy. We then develop effective identification signatures of the two segments according to Benford's law and power law. In our findings, there exists a bidder class, namely, individual bidders who confirm the distributional pattern of Benford's law. They tend to focus on the current auction and actively participate in the price formation processes. On the other hand, institutional bidders monitor entire market, which comprises multiple overlapping auctions, and reveal tactical bid shading behavior. Due to the negative impact on the seller-side gain, the forensic accounting literature categorizes this type of agent behavior as a bidder-side fraud [13]. The contribution of this paper can be recognized in development of effective and efficient methods to identify and detect such transaction patterns.

Our study is broadly based on the emerging trend on web based online auctions and associated bidder behavior. Auction participants are spatially and temporally dispersed but they are virtually connected. This dynamic pricing schemes in online auctions especially IT driven proxy bidding systems, affects the nature of agent behavior and, in turn, the earnings in the market. It has been pointed out that, unlike the case of physical auctions in traditional format [16][20] which relies on the Bayesian-Nash equilibrium solution space, research on online auctions tends to be more complicated in a way that the same solution is not easily generalized to competing auction cases and sometimes there exist multiple equilibriums [4][5][6][7].

This complexity of current online auctions motivates experiments of a wide range of bidding strategies to maximize entire market-level gains. One of these strategies is market monitoring and institutional bidding, where bidders move around multiple concurrent or overlapping auctions, bid on the lowest price progress and eventually they attempt to lower the ending prices [1][4][18][19]. This strategic group typically appears in B2C cases of highly liquid online auction market. The advantage of institutional bidding is explained in terms of winning probability and the price gain [4]. Since this type of bidding strategy is nicely supported by online auction features such as proxy bidding systems, in which a bidder can submit his/her willingness-to-pay then an automatic bidding agent places bids according to the price progress, monitoring the entire market and bidding on multiple auctions can be done at relatively low cost [9].

This capability is not well addressed in traditional auction formats that assume independence and isolation of auctions and the gap between traditional settings and current online format creates possibility of a newly emerging online auction fraud.

After the passage of Sarbanes-Oxley Act of 2002, there has been a high demand for fraud prevention and detection. The information asymmetry between individual and institutional bidders can incur tactical bid shading and some literature point out that this type of behavior is categorized to online auction fraud [13].

While majority of recent studies are focused on the economic impact of overlapping auctions, there has been lack of research on the identification and detection of such kind of strategic behavior. We are motivated to address the issue in the following research objectives.

- We characterize heterogeneous strategic agent segments using k-means clustering approach
- We find identification signatures of the clusters. Difference in the signature patterns is interpreted in terms of the deviation from Benford's Law and Power Law
- Surplus of the clusters, which impose direct impact on the market earning is illustrated.

Bidder profiling based on correct identification of unique signature is closely related to the overall price determination process and market earnings. Figure 1 indicates that for identifying and preventing institutional bid-shading behavior, there must be comprehensive understanding on overlapping auction market which governs individual mechanisms. The causal impact of the market on the individual mechanisms establishes heterogeneous set of bidder types. This in turn addresses necessary environment to uncover institutional bidding as well as individual naïve bidding activities. We posit that IT based online auctions and automatic bidding agents have a relatively short history of studies than traditional physical settings and the regulators of current online auction market do not have a significant role in preventing or detecting fraudulent bidding patterns. The main reason for this is the lack of control for bidder entry and bid placement in a course of an auction. However, as the cost of bidding [9] generates endogeneity to the market condition [14], the overlap of auctions, more specifically, the degree of overlap across multiple auctions influences bidders' strategic space.

We identify institutional bidders and individual bidders based on the 'number of auctions, they step in. Depending on market supply condition, this variable is a good indicator for individual or institutional demand as well as potential market monitoring [4].

Then we formulate a set of hypotheses to compare the two distinct patterns and develop identification signatures in terms of 'bid intensity' which is an effective differentiator for recognition of institutional and individual bidders. Benford's law and Power law are used for rigorous validation of the proposed hypotheses.

We find a significant difference between institutional bidders and individual bidders in bid intensity. As in [3][4][6], the entry decision and bidding pattern of bidders are endogenous to mechanism design factors as well as overall market supplies. The interplay of heterogeneous strategies alters the market efficiency, agent surplus and the auctioneer's gain. The rationale for using the bid intensity as the indicator in our paper is that, due to the multi-auction monitoring behavior of institutional bidders, they tend to decrease the bidding cost [4][9]. Their bid frequency is low to minimize the time they spend in a particular auction whereas high bidding cost through ratchet bidding pattern occurs in case of individual bidders. Bid intensity can be a good indicating variable to differentiate two agent groups.

An innovative way to compare the two groups in bid intensity is developed. We considered the digital distribution of bid frequencies. As highlighted in [8][17], the Benford's law based on the digital distribution is a useful characterization method of accounting data. Both institutional and individual bidders show the highest frequencies on the first digit, and it decreases exponentially. Interestingly the bid intensity distribution of individual bidders is similar to Benford's law while institutional bidders show a steeper decrease in the digital distribution. It is a strong indication that the digital distribution of bid intensity is an effective and also efficient identifying signature to differentiate institutional and individual bidders.

Our results provide important theoretical and practical implications. From a theoretical perspective, they can help us understand the nature of heterogeneous online auction bidding strategies and the identification of unique signatures to represent the own characteristics. From a practical perspective, the outcomes of this study will be of interest to practitioners who investigate dynamic pricing models, audit revenue cycles of online auction markets and develop optimal online bidding systems.

While studies on concurrent auctions provide some evidence of correlation between overlapping auctions and institutional bidding, they do not address the core issues of forensic accounting such as characterization and identification of agent signature, potential fraud and economic consequences to the market earnings. Significant inference can be derived from institutional bidding activities. Based on their market monitoring behavior, it is likely to be involved to bid shading which is categorized to a bidder side fraud [13]. We attempt to explore these issues by identification of bidder signature, validation our proxy by Benford's Law and simulation model illustration.

Our paper organizes as follows. The next section provides a brief overview of overlapping online auction market. The third section presents our model and hypotheses related to institutional and individual bidder segmentation by *k*-means clustering and identification of signatures. In the fourth section, we discuss our findings regarding the implication of bidding strategies to the market earnings.

2. Market characteristic and data

1236 instances of overlapping online auctions at Sam's Club are used for this study. We collected data about full bidding histories on various electronic goods from Samsclub.com. High degree of market liquidity is well reflected in the form of hundreds of concurrent and overlapping listings to provide good environment for our study of institutional bidding phenomenon. We used automated *PHP* scripts to download listing numbers, product details, opening and closing time as well as full bidding records into a *MySQL* database.

Examination of simultaneous and overlapping auctions is based on the formalization of Bapna et al. (2009). As in the following condition, two auctions are defined as overlapping auctions when they share a non-zero amount of time span.

$$\boldsymbol{l_i} + \boldsymbol{l_j} \geq \max[\boldsymbol{t_i^c}, \boldsymbol{t_j^c}] - \boldsymbol{min}[\boldsymbol{t_i^o}, \boldsymbol{t_j^o}] \quad \textbf{(1)}$$

where t_i^o: opening time of auction *i*
t_i^c: closing time of auction *i*
l_i : the length of auction *i*, t_i^c - t_i^o

In our sample, average auction duration is 1.02 days and bid increment is $1. Bids are submitted by proxy bidding system in which bidders can enter their maximum willingness to pay then the automatic bidding agent increases bid according to the auction progress. All the items are brand-new. Since the only auctioneer (e.g., Sam's Club) manages all the items in English auction format, the site is ideal place to examine the nature of multiple overlapping online auctions and the optimal market design strategy.

Figure 1 shows 100 instances of overlapping online auctions for an identical item. The opening time, closing time and duration along with the total number of bids made are presented in Table 1.

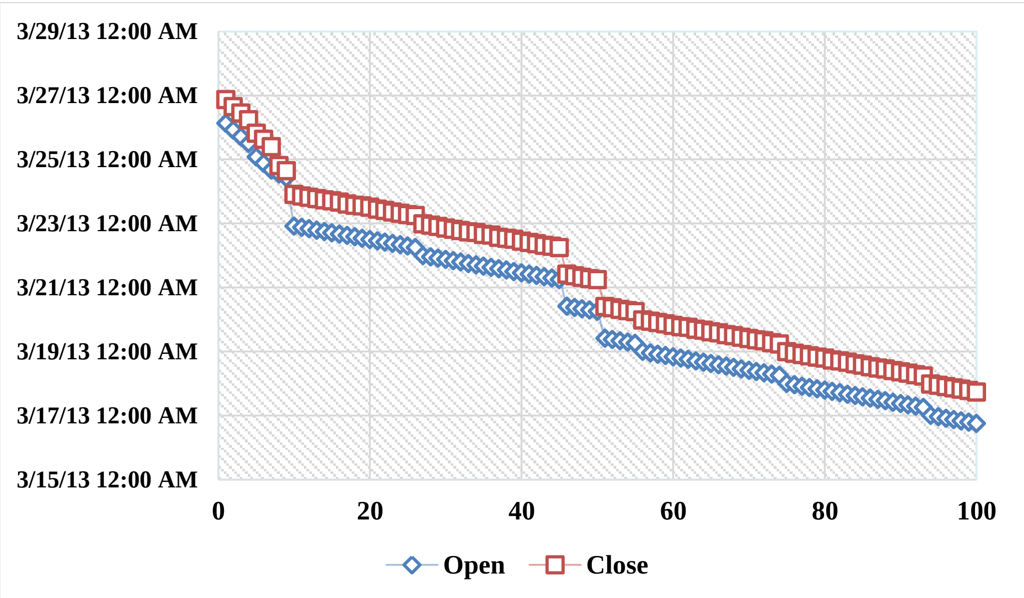

Figure 1: 100 overlapping auctions of an identical good

	Open	Close	Duration (Days)	Number of Bids
Mean	3/8/13 2:20	3/9/13 2:48	1.02	11.39
Standard Error	n/a	n/a	0.01	0.22
Median	3/5/13 14:10	3/7/13 12:45	0.99	10.00
Mode	3/3/13 15:00	3/13/13 10:00	0.99	9.00
Stand. Dev.	n/a	n/a	0.19	7.67
Sample Variance	n/a	n/a	0.03	58.76
Kurtosis	n/a	n/a	26.41	10.48
Skewness	n/a	n/a	4.82	2.45
Range	n/a	n/a	2.27	75.00
Minimum	2/24/13 15:00	2/25/13 18:50	0.23	1.00
Maximum	3/26/13 3:00	3/26/13 21:30	2.50	76.00
Count	1236	1236	1236	1236

Table 1. Sams Club Auctions

3. Model and Hypotheses

3.1. Characterization of individual/ institutional bidders

In characterization of bidding strategy, significant inference can be derived from institutional bidding and market monitoring activities. The nature of multi-auction monitoring involves a possibility of bid shading. As Jenamani et al. [13] suggested bid shading is categorized to a buyer side fraud and this is often motivated by high market liquidity, which is well reflected in our study as high degree of overlapping or simultaneous auctions.

We measured cross bidding activity with Number of Overlapping Auctions (NOA) the bidder participated in Number of Items (NOI) the bidder bid on during the time period. According to Bapna et al. [4], these two metrics depict cross bidding activities that depend on the market condition. To effectively differentiate the bidding strategy, we use *k*-means clustering algorithm.

Given a set of observations ($x_1, x_2, \ldots, x_n$), where each observation is a multi-dimensional real vector, *k*-means clustering aims to partition the n observations into *k* sets of clusters: $S_1, S_2, \ldots, S_k$ where $k \leq n$ to minimize the within cluster sum of squares of distances between data points and the cluster centroid. The algorithm is represented in the following optimization problem.

$$\min \sum_{i=1}^{k} \sum_{x_i \in S_i} |x_i - \mu_i|^2 \quad (2)$$

where μ_i is the cluster centroid of S_i.

Our 2-means clustering clearly differentiates institutional bidders from individual bidders in both NOA and NOI (see Table 2). While both clusters have similar inter cluster distances as in Table 3, a considerably smaller portion of bidder shows institutional bidding strategy.

Cluster	NOA	NOI
Individual Bidders	2.244	1.163
Institutional Bidders	19.685	3.925

Table 2. Cluster centers

Inter cluster distance	Individual Bidders	Institutional Bidders
Individual Bidders	0	17.658
Institutional Bidders	17.658	0

Table 3. Inter cluster distance

Cluster	Number of Observations	Average Distance in Cluster
Individual Bidders	2249	0.320
Institutional Bidders	240	2.129
Overall	2489	0.495

Table 4. Data summary

3.2. Bidder signature based on Benford's law

An essential requirement for institutional bidding is the simultaneous occurrence of multiple auctions. The extent to which multiple auctions of the same product are simultaneously available is termed here as market liquidity. This is a market condition which consists of multiple overlapping supply sources. The overall market earning, in this case, is not just the outcome of bidding games in independent auctions but the interplay of multiple auctions as well as the bidding behavior. The nature of high liquidity in today's ecommerce systems engenders institutional bidding pattern in a new domain where the institutional bidders have opportunity to monitor multiple auctions in a given time span, mover around the sources and choose auctions which maximize their expected return.

In fact, this type of strategy can be well supported by the proxy bidding system in Sam's Club which facilities automatic bid placements. Bidders simply specify their maximum willingness to pay then the automatic bidding agent increases the bid according to the auction progress. Bidders can adjust their willingness-to-pay in the course of an auction. Then the bidding procedure will change accordingly. Our dataset indicates that both institutional bidders and individual bidders utilize this proxy bidding system. While bidders adjust their bids, majority of bidders place just a single bid without any adjustment. The distribution of the bid adjustment is well characterized by exponentially decreasing curves starting from the low digit values. The institutional bidders, however, tend to show steeper decrease in the distribution of bid-adjustment implying they adjust bids even less frequently than individual bidders. For our signature analysis, we introduce a variable, number of bid adjustments in an auction (NOB) to capture this distributional pattern and formulate the following hypothesis.

H1: In the combined bidder pool (i.e., institutional and individual bidders), lower digit values appear more frequently than high digit values for the number of bids.

Compared to individual bidders, institutional bidders do not place upfront proxy bids. They tend to arrive at the later stage of an auction and show less frequent bid-adjustment. This can be related to their entire market monitoring process which potentially limits the amount of time they can spend in individual auctions for bid adjustment. Therefore, we propose the number of bid adjustment (NOB) made by a bidder in a given auction as the identifying signature of institutional bidding activity, which is expected to have a negative correlation with the tendency of intuitional bidding.

H2: The number of bid adjustment (NOB) is negatively related to the tendency of institutional bidder in an auction.

Unlike the institutional bidders, individual bidders stay in an auction for a relatively longer time. They tend to focus on the current auction without market monitoring. Table 2 shows that number of auctions a bidder participates in (NOA) and the number of items (NOI) of individual bidders are significantly lower and this allows frequent bid adjustment (NOB). It implies that individual bidders have motivation to participate in the bid progress and this is reflected by high bid adjustment frequency (NOB). The distribution of NOB of individual bidders is likely to have a longer tail than that of institutional bidders.

H3: the digital distribution of bid frequency (NOB) of individual bidders decreases less steeply than that of institutional bidders.

Analytical procedures based on digital analysis have long been used in the area of forensic examination. Benford's law is one popular form of digital analysis which examines entire account to see if the numbers fall into the expected distribution (Durtschi et al. 2004). While it is known that most accounting related data is expected to conform to Benford's law, there has been lack of attention to the dynamic pricing mechanisms such as auctions. Especially, the complexity of current online auction market extends the research scope from an isolated single auction to the network of overlapping auctions and enhances the significance of earning estimation. Given that institutional bidding entails substantial revenue loss to the seller (Bapna et al. 2009), the early-stage characterization and identification of the bidder type will help optimal choice of market format and liquidation schedule.

It is important to note that the bid frequency, NOB, is an effective identifier of bid shading which is categorized to a buyer-side fraud (Jenamani et al. 2007) and can undermine the seller surplus. The market monitoring of institutional bidders provides a certain level of information-advantage regarding the price formation structure and this motivates the bid-shading of institutional bidders and, in turn, lower ex-ante price of the market.

Digit	Frequency
1	0.301
2	0.176
3	0.125
4	0.097
5	0.079
6	0.067
7	0.058
8	0.051
9	0.046

Table 4. Digital Distribution in Benford's Law

We compared the digital distribution of bid-adjustment-frequency (NOB) with the Benford's distribution (Table 5) for combined bidder pool, institutional bidders, and individual bidders. The overall tastings examine the goodness of fit of observed patterns to the Benford's distribution using χ^2 method. In Figure 1, our baseline distribution is the combined bidding activities which include institutional bidders and individual bidders. The NOB is measured as the digital (1-9) frequencies. Note that the unit of analysis is the market which comprises a set of overlapping auctions which motivates both individual and institutional bidding. In Figure 2 and Figure 3, the baseline distributions are the NOB patterns of institutional bidders and individual bidders respectively.

In general actual frequency distributions of NOB sample follow the exponential power function.

$$\boldsymbol{f = c_1 \cdot e^{-c_2 \cdot d}} \qquad \textbf{(3)}$$

where d: digit value (= 0 to 9)

c_1 and c_2: scaling constants.

Our sample shows that the difference of two groups (i.e., individual and institutional bidders) in the behavioral pattern is significantly different. Institutional bidders deviate significantly from the pattern of combined distribution. It reaffirms that the bid adjustment frequency (NOB) is an effective indicator that represents the signature of two groups.

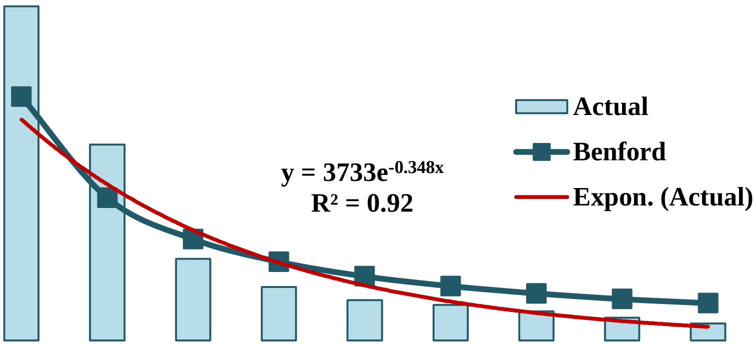

Figure 2. Digital Distribution: Actual vs. Benford's

(Chi-Sq = 84.378 + 84.361 + 50.012 + 50.002 + 12.412 + 12.409 + 28.734 + 28.728 + 31.635 + 31.628 + 23.152 + 23.147 + 24.936 + 24.931 + 31.573 + 31.567 + 46.060 + 46.050 = 665.715, DF = 8, P-Value = 0.000)

Figure 2 is tested using a χ2 method. The first hypothesis (H1) is tested using the model that specifies the NOB of combined bidder pool as the actual distribution and the Benford's law distribution as the benchmark. It is not confirmed that the actual distribution follows Benford's law (χ^2 = 665.7 with p-value of 0.000). A closer examination revealed that the digital distribution of overall bids shows steeper decrease than the Benford's distribution and is well characterized by the exponential function $y = 3733 \cdot e^{-0.348x}$ with R^2 of 0.92.

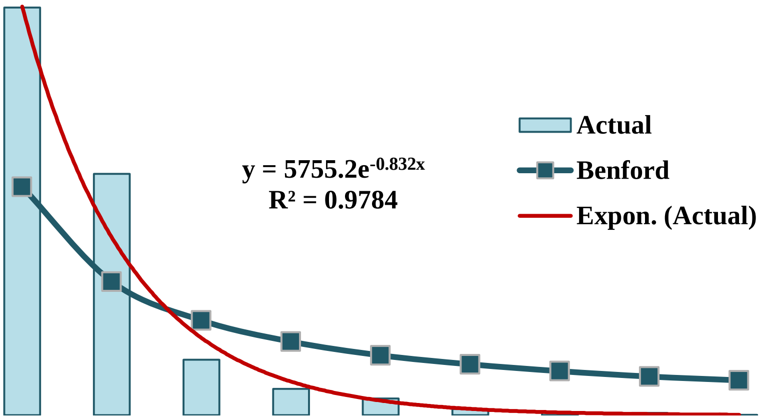

Figure 3. Digital Distribution: Actual Institutional vs. Benford's

(Chi-Sq =154.402 +154.402 + 94.778 + 94.778 + 31.531 + 31.531 + 68.709 + 68.709 + 74.145 + 74.145 + 107.251 +107.251 + 111.514 +111.514 + 99.062 + 99.062 + 104.037 +104.037 = 1690.859, DF = 8, P-Value = 0.000)

The results of the comparison model for the institutional bidders are presented in Figure 3. The difference between institutional bidding pattern and Benford's distribution is statistically significant (χ^2 value = 1690.9 with p = 0.000). Although both NOB of the combined bidder pool and institutional bidders are negatively correlated to the digital values, institutional bidding pattern shows a much shorter right tail than the combined pool as well as Benford's distribution, providing empirical support for the hypothesis H2.

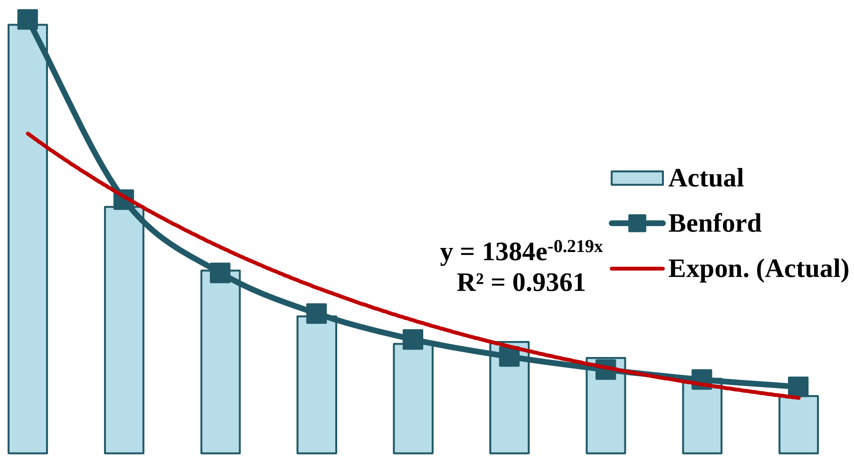

Figure 4. Individual DF = 8, P-Value = 0.299

(Chi-Sq = 0.060 + 0.060 + 0.180 + 0.180 + 0.025 + 0.025 + 0.052 + 0.052 + 0.165 + 0.165 + 1.799 + 1.799 + 1.286 + 1.286 + 0.009 + 0.009 + 1.191 + 1.191 = 9.533, DF = 8, P-Value = 0.299)

Unlike the other two cases, the χ^2 analysis indicates that there is no statistically meaningful difference between the NOB of individual bidders and the Benford's distribution (χ^2 value = 1690.9 with p = 0.000). The hypothesis H3 is supported that the NOB distribution of individual bidding pattern decreases exponentially, and the slope of the decrease is more moderate compared to the case of institutional bidders. Furthermore, the distribution of individual bidders in fact follows the pattern Benford distribution and effectively identifiable by this signature.

We can see that when there is a bidding pattern regarding the bid-adjustment frequency (NOB) which is statistically equivalent to Benford distribution, it can be translated to an indication of individual bidding. Similarly, when there were bidding patterns which deviate significantly from Benford distribution, it is likely to be institutional bidders. For more specific characterization, we traced three exponential power functions which characterize the distributional patterns to identify unique signatures.

4. Conclusion

Relying on the formalization of Bapna et al. (2009), we examined effective characterizing signatures of institutional bidders and individual bidders in overlapping online auction market. Since these two strategic clusters play crucial roles in dynamic pricing mechanisms, the early-stage characterization and identification of behavioral patterns imposes significant revenue impact on the market. Our simulation analysis validates this for various combinations of the bidder clusters.

In this paper, we demonstrated existence of unique signatures of bidder strategies in the context of bidder-side fraud prevention/detection in agent-based online auctions. We provide a framework which describes how the bidder profile and identification of strategic signature are related to the overall market earning. It should be highlighted that identification of bidder signature is beneficial to those who are interested to optimize the liquidation schedule of overlapping resources.

We simulate the importance of proportional mixture of institutional and individual bidder to the market revenue. Since this analysis establishes test bed for a set of auctions which accommodate strategic mixture of two bidder types, it provides practical revenue implications based on identification and detection of bidder signature.

From a practitioner's perspective, this will help detect the bidding patterns and optimize the market format. Institutional bidders and individual bidders are significantly different in the digital distribution of 'bid intensity'. Interestingly the distribution of individual bidder follows the Benford's law implying that it can be an effective identification benchmark for high revenue generating and non-fraudulent agents. Institutional bidders tend to minimize the bid frequency in a particular auction and the exponential power distribution of the bid intensity decreases faster than that of individual bidders.

We performed data-partition to institutional clusters and individual clusters using *k*-means clustering method in which number of auctions an agent bids in (NOA) and number of item categories (NOI) have been used to characterizing attributes. The rational is that these two variables reflect market liquidity at a given point of time and, hence, capture the environment where institutional bidding occurs. Institutional bidders are sensitive to the bidding cost in a particular auction and minimize the amount of time they use in a single auction. Our analysis shows that as the number of simultaneous auctions increased, more bidders are engaged in institutional bidding this might be an indirect explanation of the negative correlation between the proportion of institutional bidders and the market earning. Our simulation analysis validates this argument and illustrates the impact.

Our results are based on a real-world online auction site. The generalizability of our findings is not necessarily limited to the auction mechanisms. As seen from Bapna et al. (2009), when multiple mechanisms exist concurrently, the revenue difference is significant depending on the degree of overlap. The explanatory power of the models can be extended to any type of mixture of non-auction post price markets. Increasing the number of selling channels and time span will lead to more simultaneous or overlapping sources and thus motivate more institutional activities across the market. Early-stage detection of user types that are highly correlated to the market earnings will provide more controlled experimental environments to test potential causes and effects of institutional bidding and resulting economic consequences.

In the context of forensic accounting, the identifying signature of institutional bidder is significantly different from individual bidders. Institutional bidders tend to place significantly less number of bid-adjustments than individual bidders. When there is high degree of overlap in the market, the time an institutional bidder can spend in a specific auction decreases and this will decrease the bid frequency. Note, however, that institutional bidders still show higher surplus, than individual bidders in the price premium, which implies their expertise to monitor the market, locate the lowest price progress auction and deliberately exercise bid shading which is categorized to an online auction fraud (Jenamani et al. 2007). Given the nature of online auction market, practitioners have limited control over a bidder's entry decision and her bidding strategy. Identifying unique bidder signature will provide good guidelines for user management and market design.

Finally, we provide suggestions to market regulators and policy makers to incorporate continuous monitoring for potential bidder side fraud and resulting revenue estimation. Apparently, the key requirement for this is correct identification of bidder signature. In addition, the underlying characterization based on strategic pattern clustering, should also be continuously updated for possible deviation from current conditions.

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