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A Resilience-based Sourcing Model with the Numerical Optimization of Suppliers under Global Supply Disruption Risks

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Abstract

As the COVID-19 has exposed the shortcomings of globalized supply chains and logistics networks, the numerical optimization of an offshore supply base with lower-risk suppliers is receiving significant attention from buying firms to consider as a practical option for resilience improvement and risk mitigation. Firms may improve their resilience performance by reducing their foreign supplier dependency and diversifying suppliers in number. However, reshaping their supply base with the increased foreign supplier redundancy may not be equally effective to every firm, and resilience improvement may not always result in greater economic returns. Therefore, business enterprises should have the ability to evaluate whether the chosen number of offshore suppliers will present the optimum conditions for the cost-effective resilience level they need to minimize supply disruption risks. To address this issue, we propose a risk-resilience-based supplier optimization model under short- and long-term contract procurement situations and assess the numerical optimization solutions against their impact on the financial performance of the buyers' offshore supply chain. Focusing on the simultaneous effect of wholesale price uncertainty and the number of suppliers held in an offshore supply chain, we provide essential conditions and properties of the resulting optimization functions with decision rules to determine the best supplier choice set that can generate resilience for risk mitigation while maximizing supply chain profitability. We further show how buying firms can make optimum-number-of-offshore-supplier decisions with supply base resilience in different supplier preference ratings and unforeseeable supply disruption circumstances

Keywords: Supply chain management; uncertainty modelling; numerical optimization; stochastic processes

1. Introduction

Following the extraordinary supply turbulence worldwide due to the impact of the COVID-19, supply chain adaptability to disruption with resilience improvement has become an inevitable necessity factor for global buying firms to mitigate risk and remain competitive in the age of growing uncertainty, especially for those practicing a lean sourcing strategy that involves a single offshore source or a small number of foreign suppliers in favor of the availability of supplies at a lower cost. Recognizing that resilience would be one of their most important metrics when evaluating the use of global procurement, business enterprises seek options to reduce their risk exposure and plan investments in developing a lower-risk sourcing model.

To cope with supply disruptions linked to geopolitical tensions, infectious diseases, natural disasters, and other unknowable global events, the concepts of supply base regionalization and resilience-building (Rugman & Verbeke 2004; Wang et al., 2010; Contractor et al., 2010; Bhamra et al., 2011; Chopra & Sodhi, 2014; Scheibe & Blackhurst, 2018) have been continuously evolving in supply risk mitigation management. However, the survey data conducted after the COVID-19 crisis reveal that there is no massive support for reshoring or relocation of sources. For instance, less than 15% of companies consider reshoring, and only 30% of companies favor nearshoring (Dib, 2020). Over 70% of respondents have no plans to relocate their sourcing outside China due to the pandemic, at least in the short term, and preferably look for new suppliers and consider increasing cost-effective foreign suppliers for resilience improvement (Di Stefano, 2021). The survey findings imply that a shift to a regionally based sourcing for resilience improvement will be partial or marginal generally because of huge costs and risks related to switching primary suppliers. Developing a regionalized supply base may bring additional investment, particularly when its location lacks supporting services, specialist suppliers, or efficient transport and communication links (Enderwick & Buckley, 2020). The regulatory burden of moving an already established supply chain to a different location can also make it difficult for them to completely regionalize their supply chain network (Hippold, 2021). Even though regionalization can reduce vulnerability, the region- or country-specific uncertainty remains at risk. In these respects, reducing supplier dependency levels by diversifying sources of supplies in optimum numbers can be a viable option for resilience improvement and risk mitigation.

The challenge, however, is how to assess the optimal level of supply chain resilience needed for buying firms to minimize the negative impact of future supply disruptions and determine the most cost-effective quantity of suppliers to support their supply continuity. To meet this challenge, we propose a quantitative sourcing optimization model with three research objectives. First, we explore the ever-increasing supply uncertainty inherent in offshore supply chains and create a systematic risk assessment framework for buyers to analyze the impact of supply disruption and measure their vulnerability. Second, considering the wholesale price volatility associated with unforeseeable global supply disruptions, we examine how increased supplier redundancy for resilience building can affect the cost per unit of procured items of preferred offshore suppliers. We then evaluate the numerical optimization solutions against their impact on the financial performance of the buyers' supply chain, including the total expected cost and supply chain profitability from managing the selected set of preferred offshore suppliers. Third, we characterize the efficacy of numerical supplier optimization for two types of supplier relationships, long- and short-term procurement contracts under wholesale price uncertainty. We derive a decision-making policy for buyers to select an optimum set of offshore suppliers in the most timely and profitable way during their procurement planning horizon. The contribution of this paper is a deeper understanding of offshore sourcing risk management and quantifying the risk-reduction effect from optimizing the number of offshore suppliers. Our model can aid decision-makers in estimating the vulnerability of their organization's current sourcing strategy and new procurement plans, enabling them to make more informed decisions about how to create a resilience buffer against unexpected global supply shocks. The proposed model also can help them compare the risk/return outputs resulting from changing (increasing or decreasing) the potential foreign sources of supplies and make a strategic decision on the optimum number of offshore suppliers that best fit their corporate objectives.

The remainder of this paper is organized as follows. Section 2 conducts a brief literature review on supply risk management, supply base optimization, and risk mitigation approaches to sourcing strategy formulation. In Section 3, we develop a global sourcing model with optimal solutions for short- and long-term contract procurement plans in the presence of the wholesale price uncertainty of offshore suppliers. Section 4 presents the numerical example analysis to illustrate how the proposed model can help supply chain managers and planners cost-effectively select the optimal number of suppliers for resilience improvement and risk mitigation. Finally, Section 5 provides a summary of our conclusion with some suggestions for future research.

2. Related literature review

Businesses have taken advantage of a global sourcing approach based on the search for capable suppliers worldwide. To stay competitive in the market, they have continuously cut off their supply chain costs by adopting unique and distinctive sourcing strategies. Buying companies, for example, are pursuing lower unit costs by switching from domestic suppliers to low-cost international suppliers, reducing the number of suppliers in their global supply base, and employing just-in-time replenishment (Christopher & Peck, 2004; Yu et al., 2009). However, the internationally extended ultra-lean sourcing method can create a greater vulnerability to supply chain disruption risks due to increased uncertainty associated with the variant-rich global supply chain processes and logistics (Poirier & Quinn, 2004; Meixell & Gargeya, 2005; Costantino & Pellegrino, 2010). Global sourcing risk mitigation is not a new concept in the supply chain management literature. However, after the recent outbreak of the CORONA-19 pandemic caused unprecedented global supply shortages of raw materials, components, and products, supply base optimization issues for resilience improvement and disruptive risk reduction are becoming more critical than ever before.

The risk refers to the probability of a negative outcome and the impact of uncertain factors (Claypool et al., 2014; Ghadge et al., 2017). Supply risk means the probable failure in supplying materials and goods in terms of time, quality, and quantity (Sreedevi & Saranga, 2017). Uncertainty refers to any situation where it is impossible to assess the likelihood of future risk events (Klibi et al., 2010). Supply uncertainty is related to the upstream uncertainty associated with materials' supply, including material price, supply capacity, supply lead time, and alternative sourcing availability (Pujawan, 2004; Chang & Lin, 2019). Supply risk and uncertainty can cause supply chain disruption in the form of unanticipated events that disturb the normal flow of goods and materials within a supply chain and, consequently, expose firms within the supply chain to operational and financial risks (Hendricks & Singhal, 2005; Craighead et al., 2007). To evaluate the impact of supply risk and uncertainty on supply chain performance and suggest optimal decision-making on preparing for the occurrence of random disruption risk, Chopra and Sodhi (2004) conceptually classified supply chain uncertainty into nine categories and discussed mitigation strategies for the risks associated with each type. The work of Simangunsong et al. (2012) showed a comprehensive list of fourteen sources of uncertainty, ten strategies to reduce the uncertainty risk, and eleven approaches to cope with uncertainty derived from the increased complexity of global supply networks. Simchi-Levi et al. (2002) and Tang (2006) discussed the strategy formulation processes to make a global supply chain more resilient in the face of disruptions. Tomlin (2006) focused on tactical plans for robust global supply chain performance highlighting the difference between mitigation tactics (actions a firm takes before disruption) and contingency tactics (actions a firm takes after a supply disruption has occurred). Kleindorfer and Saad (2005) identified three overarching categories that could be the primary sources of supply chain disruption risk. They included operational contingencies (e.g., equipment malfunctions and systemic failures), abrupt discontinuity of supplies (e.g., labor strikes and natural hazards), and terrorism or political instability.

In the era of global competition, businesses are now focusing more on improving offshore sourcing resilience and configuring a risk-responsive supply base to derive a competitive edge (Nelson, 2013; Dib, 2020; Di Stefano, 2021; Hippold, 2021). Many authors (Zeng, 1998; Tomlin, 2006; Burke et al., 2007; Fang et al., 2013; Buckley, 2020) have framed the debate on how to select sources of supply for risk mitigation and how many suppliers to retain in the supply base for supply continuity and effective business operations. Considering the impacts of supply disruption risks on supplier selection under various situational settings, they discussed the choice between single, soul, dual, multiple, and contingent sourcing strategies. Proponents of a single-source procurement strategy argued that monogamist supplier relationships eliminate the cost of maintaining multi suppliers (Costantino & Pellegrino, 2010). Highlighting the importance of a buyer-supplier partnership in procurement management, Larson and Kulchitsky (1998) suggested that a single source with supplier certification can offer higher quality at a lower total cost to a buyer. Nam et al. (2011) pointed out that the pursuit for better quality, faster processing, and closer collaboration has led firms to favor the use of fewer suppliers. Using a US case study, Azad and Hassini (2019) found that a dynamic pricing recovery strategy is more efficient in single-sourcing networks than multi-sourcing networks.

Some studies, however, discussed that reducing the supply base leaves a firm with fewer options when faced with the risk of supply chain disruption, pointing out that a multiple-sourcing strategy can react more effectively to unexpected events. The study of Bakos and Brynjolfsson (1993 a, b) showed that a multi-sourcing model could help firms broaden the range of their options and increase their bargaining power over wholesale price, which could lead to cost savings. Wang et al. (2010) focused on identifying the circumstances that favor a particular sourcing strategy rather than determining which type of sourcing strategy was inherently superior. The findings of their study showed that dual sourcing would be a better strategy if the supplier reliability heterogeneity is high. Sawik (2014) proposed a comparative analysis model to determine the optimal solutions for a sourcing strategy. He discovered that if there existed a high level of supplier reliability, the impact of disruption risks could be mitigated by selecting more suppliers. The downside of a multiple-source procurement strategy is the increased costs for managing contracts with more than one supplier (Costantino & Pellegrino, 2010). Although firms involved in international arrangements of sources can achieve supply chain efficiency by reducing suppliers in their supply base, a lean sourcing model requiring a high degree of dependence on few suppliers can also be risky and disadvantageous for them with poor resilience in dealing with unexpected supply disruptions (Talluri & Narasimhan, 2005). Therefore, the buying firms need to evaluate the tradeoff between low cost with unmanaged supply risk and high cost with managed supply risk (Chakravarty, 2013) and select the optimal number of suppliers to reduce the adverse effect of disruption with cost as minimum as possible. With the ongoing evolutions in information technology (IT), researchers have explored how technologies can function in the performance improvement of global supply chains. Bakos (1997) illustrated that buyers could increase the number of suppliers by effectively reducing supplier searching costs. The study of Liker and Choi (2004) showed that IT-based sourcing could help buyers build a long-term collaborative partnership with suppliers and reduce their costs by obtaining a smaller number of suppliers. Aral et al. (2018) examined the IT's effects on supplier relationships using actual industry data. Their empirical findings suggested that greater physical capital intensity could increase the level of required assurances to suppliers by contacting fewer suppliers or entering longer-term relationships.

Several notable contributions to the research on an optimal supply base have used the decision-tree method to determine the optimal number of suppliers considering the risk of supplier failure. The work of Berger et al. (2004) proposed a decision-tree model to find the optimal number of suppliers considering two states of nature. Ruiz-Torres and Mahmoodib (2007) extended the work of Berger et al. (2004) by adding individual supplier failure to their decision-tree model. Their work showed that the optimal number of suppliers increases with higher individual unique-event failure probabilities. However, when individual unique-event failure probabilities are low, a single-sourcing strategy is the lowest cost option. In addition to the decision tree approach, some studies have used other means to determine the optimal supplier base. Weber et al. (2000) developed a model that determines the optimal number of suppliers by utilizing multi-objective programming and data envelopment techniques. Kauffman and Leszczyc (2005) considered two factors of buyer utility and decision-related cost to derive the optimal size of a supplier choice set for both one-time and repeated purchases. To incorporate uncertainty issues in optimization models, researchers have utilized stochastic programming approaches. Fine and Freund (1990) and MirHassani et al. (2000) proposed stochastic models for capacity planning. Others proffered stochastic models for the optimization of supplier selection and supply chain network design (Santoso et al., 2005; Azaron et al., 2008; Bilsel & Ravindran, 2011; Li & Zabinsky, 2011; Nam et al., 2011; Babich et al., 2012; Baghalian et al., 2013).

There have been significant research efforts to understand the impact of supply risk and uncertainty on sourcing strategy selection. However, few researchers have demonstrated the analytical framework to quantify the benefits of the numerical optimization of offshore suppliers to build supply base resilience. In this paper, we attempt to develop a quantitative decision-making model to present our insights on how buyers can formulate resilience strategies and manage offshore supply base with the optimum number of suppliers under global disruption risks.

3. Model

We now develop a modeling framework to analyze and quantify the impact of an offshore sourcing strategy comprised of an optimal number of suppliers on supply chain resilience and risk mitigation. We consider a buying firm searching for capable international suppliers to acquire raw materials or product components over its planned sourcing horizons. We assume that the firm can identify multiple potential sources worldwide to consider and create a preference list containing n eligible offshore suppliers who can meet its business needs. We also assume that offshore sourcing of supplies will inevitably face more risk and uncertainties when it spans multiple countries and internationally enlengthens supply networks and logistics. As discussed in Section 2, risk and uncertainty are distinguishable in a mathematical concept. Supply risk refers to a random event with knowable probabilities, and supply uncertainty refers to an unexpected event with unknowable probabilities. For simplicity purposes, we use the term supplier uncertainty to encompass supply risk and supply uncertainty. We reflect the effects of supplier uncertainty in our optimization modeling processes and illustrate how the supplier uncertainty can influence wholesale pricing among n eligible suppliers in the preference list. A supply base strategy designed to prepare for and deal with unforeseen and unexpected disruptive incidents may require an upfront investment in risk mitigation, which can cause a higher cost of sourcing supplies. Thus, we investigate the trade-off effects between the resilience improvement cost for risk mitigation and the financial effectiveness of the chosen number of suppliers. The objective of our model is to identify the optimum number (n*) of offshore supply partners for the buying firm to retain in its offshore supply base to build resilience and mitigate supply disruption while maximizing supply chain surplus.

3.1 Supplier uncertainty associated with the n offshore suppliers in the preference list

Given the internationally extended nature of its sourcing model and depending on the level of dependency on its offshore suppliers, the buying firm may have to encounter the risk of disruptions caused by not only the internal factors within the firm's operational walls but also the external factors beyond the firm's control, such as suppliers' manufacturing flexibility and global pandemics. To cope with those uncertainties and manage supply continuity, the buying firm must have the ability to evaluate the potential sources of risk and the impacts of supply disruption on its operations and create a plan for how to respond to and quickly correct them. Even though it is difficult and intricate to identify all the factors of supplier uncertainty and accurately measure hypothetical conditions, the firm should make a scientific supplier uncertainty assessment to effectively deal with supplier uncertainty and formulate a risk mitigation policy.

In this section, we assess aggregate supplier uncertainty by utilizing a weighted factor scoring method. The buying firm first determines supplier uncertainty evaluation categories to which it must pay attention while finding and selecting offshore suppliers (e.g., capacity risk, quality risk, financial risk, and location risk with associated political conditions, etc.). Next, the firm identifies the risk factors that make up each category. Output variability and lead time variability could be examples of the risk parameters in the capacity category. The third is to evaluate each supplier against the identified risk factors' uncertainty dimensions of likelihood, severity, and velocity and then assign them a weight that reflects their relative importance and magnitude on a scale of 0.0 to 1.0. A higher weight score indicates a higher potential of supply disruption associated with the risk factor. Fourth, the firm creates a scoring system that defines the significance and contribution of each identified risk factor to the level of supplier uncertainty on a measurement scale.

Let i refer to a particular offshore supplier identified by the firm ($i = 1^{st}, 2^{nd}, \ldots n^{th}$) based on its supplier selection criteria. If the firm decides n = 3, there are three suppliers on the list. Let j refer to a risk factor that increases the likelihood of supply failure for the buying firm. Suppose that the buying firm identifies k types of risk factors (j = 1, 2, …, k) and assigns weight to each risk factor according to its measurement scale established in the fourth step described above. Then, the supplier i's composite uncertainty risk level, $r_i(t)$, based on the given weights of k risk factors, can be expressed as follows:
$r_i(t) = \sum_{j=1}^{k} w_j r_{ij}(t)$, where $0 \le r_{ij}(t) \le 1$ and $\sum_{j=1}^{k} w_j = 1$. If $r_{ij}(t) = 0$, it implies that the supplier has no supplier uncertainty regarding risk factor j. On the other hand, $\{r_{ij}(t) = 1\}$ indicates that the supplier's uncertainty level for risk factor j is exceptionally high.

After n suppliers who best fit its selection criteria are identified and included in the preference list, the buying firm evaluates the n suppliers against its established risk measurement standards. By measuring the composite uncertainty level, $r_i(t)$, for each supplier in the list, the firm ranks them in order of the lowest risk. For example, the n^{th} offshore supplier in the list would be the least prioritized one to the buying firm due to the highest risk score among the *n* offshore suppliers. Let $\sum_{i=1}^{n} r_i(t)/n = \bar{r}^n$ refers to the average level of supplier uncertainty associated with the *n* offshore suppliers. If the buying firm's decision on its supply base management is to pursue a double sourcing strategy, it should select the lowest and the second-lowest composite uncertainty score suppliers. Therefore, the value of $\bar{r}^n$ is characterized as a non-decreasing function when the number of suppliers increases such that $\bar{r}^n \leq \bar{r}^{(n+1)}$.

3.2 Optimizing the n preferred offshore suppliers

We now examine how supplier quantity and wholesale price uncertainty together can affect supply chain performance and evaluate the total cost and profitability of a multi-source model coordinated with *n* number of preferred offshore suppliers. We then derive essential conditions and properties of the resulting optimization functions to determine the best supplier choice set that generates resilience for risk mitigation while maximizing supply chain profitability.

3.2.1 Modeling the stochastic process of average wholesale price under supplier uncertainty

Different suppliers may have different wholesale prices for the same procurement items and services. Accordingly, there are opportunities for the buyer to lower its purchasing price by taking advantage of price competition between multiple suppliers and reducing the suppliers' opportunistic wholesale price risk (Trevelen & Schweikhart, 1988; Norrman & Jansson, 2004; Manuj & Mentzer, 2008; Su & Levina, 2011). For our model, we assume that a larger pool of suppliers can provide the buyer with more bargaining power from the price bidding competition between the *n* suppliers in the list (if $n \geq 2$). We then investigate the impact of acquiring *n* number of suppliers on supply chain financial performance in the presence of the wholesale price volatility risk generated by various market changes and random disruptive global events (Carter et al., 2011; Tazelaar & Snijders, 2012; Zsidisin & Hartley, 2012; Gaudenzi et al., 2018).

Geometric Brownian motion (GBM) is known for modeling the time-evolution of an asset price. Some studies used the GBM model to describe stock price paths (Shakila et al., 2017). Others used it to forecast the stochastic price paths of commodity products such as oil, petroleum product, and natural gas (Chan & Grant, 2016; Ibrahim et al., 2021); rice and coffee (Nguyen & Tran, 2015); rubber (Ibrahim, 2016); gold (Ramos et al., 2019); and iron ores (Hamdan et al., 2020). Utilizing the GBM and its applications discussed in the studies of Dixit & Pindyck (1994) and Li & Kouvelis (1998), we examine how supplier uncertainty risk can influence the wholesale prices of the offshore suppliers included in the preference list. We assume that the industry wholesale price will evolve according to the Geometric Brownian motion (GMB) with drift (μ), volatility (σ), and Wiener process (dW) that satisfies the following stochastic differential equation: $dS = \mu S\,dt + \sigma S dW$. Based on this assumption, we model the time-evolution of the average wholesale price per unit of *n* number of offshore suppliers in the preference list. If there is no confusion, we omit the time notation of *t* in equations such that $n(t) = n$ and $S\{n(t)\} = S(n)$. Let $S\{n(t_0)\} = S_0$ be the average wholesale price of the *n* offshore suppliers at the initial time. Let $E[S(n)] = M(n)$ and $E[S^2(n)] = G(n)$. One of the major reasons for offshore sourcing is the lower price of materials or components with abundant quantity available in overseas countries. We assume that the wholesale prices of offshore suppliers included in the buyer's preference list should be lower than (or at most the same with) those of domestic suppliers. Thus, we consider both the drift and volatility of domestic wholesale prices and the supply uncertainty associated with the *n* offshore suppliers in the list and then derive the function of the average wholesale price per unit of the *n* offshore suppliers. Let μ be a drift term of the average domestic wholesale price and σ be its volatility term. We define the domestic average wholesale price as $dS = \mu S\,dt + \sigma S dW$. Let the drift term and the volatility term for the average wholesale price per unit of *n* number of offshore suppliers in the list by $R_1(n)$ and $R_2(n)$, respectively. We define $R_1(n)$ as a drift term of the average wholesale price of *n* offshore suppliers and $R_2(n)$ as a volatility term of the average wholesale price of *n* offshore suppliers. With the drift and volatility of wholesale pricing under supplier uncertainty, we define the average wholesale price per unit of *n* offshore suppliers at time *t*, $S(n) = S$, as follows:

$$dS = R_1(n)S\,dt + R_2(n)S(n)dW \tag{1}$$

If both μ and σ are known, the drift and volatility terms of *n* number of offshore suppliers can be expressed as the function of μ, σ, and $\bar{r}^n$ such that $R_1(n) = f_1(\mu, \bar{r}^n)$ and $R_2(n) = f_2(\sigma, \bar{r}^n)$. We will show the parameter estimations regarding μ and σ later in the numerical analysis section.

Proposition 1.

$$\boldsymbol{S(n) = S = S_0 e^{\left[\int_0^t \{R_1(n) - R_2(n)^2/2\}ds + \int_0^t R_2(n)dW\right]}} \tag{2}$$

Proof. Let $F(S) = \ln(S)$ and $S(t_0) = S_0$, then $dF = (dF/dS)dS + (1/2)(d^2F/dS^2)dS^2$ by Ito's Lemma. Then, $dF = (dF/dS)\{R_1(n)S\,dt + R_2(n)SdW\} + (1/2)(d^2F/dS^2)\{\{R_1(n)S\,dt + R_2(n)SdW\}^2$. Since $dW \cdot dW = dW \cdot$

$dt = dt \cdot dW = 0, dF/dS = (1/S)$, and $d^2F/dS^2 = -(1/S^2)$, $dF = \{R_1(n) - R_2(n)^2/2\}dt + R_2(n)dW$. Therefore, $S(n) = S_0 e^{\left[\int_0^t \{R_1(n) - R_2(n)^2/2\}ds + \int_0^t R_2(n)dW\right]}$.

Proposition 2.

$$E[S(n)] = M(n) = S_0 e^{[\int_{t_0}^t R_1(n)ds]} \quad (3)$$

$$E[S^2(n)] = G(n) = S_0^2 e^{\left[\int_{t_0}^t \{2R_1(n) + R_2(n)^2\}ds\right]} \quad (4)$$

Proof. Since $M(n)$ is a unique solution for the differential equation of $\dot{M}$=$dM/dt = R_1(n)M$ where initial $M(0) = S_0$, $M(n) = S_0 \exp[\int_{t_0}^t R_1(n)ds]$. Because $G(n)$ is a unique solution for the linear differential equation of $\dot{G}$= $dG/dt = \{2R_1(n) + R_2(n)^2\}G$ where an initial $G(0) = S_0^2$, $G(n) = S_0^2 exp\left[\int_{t_0}^t \{2R_1(n) + R_2(n)^2\}ds\right]$ (Arnold,1974).

3.2.2 Supply chain management cost under n number of offshore suppliers in the supply base

In this section, we consider the total supply chain management cost to handle *n* number of offshore suppliers in the supply base that includes transaction cost (TRC), supplier responsiveness cost (RPC), supplier innovation cost (IVC), and logistics cost (LGC). Assume that all the *n* offshore suppliers in the list can meet an order quantity and quality requirements specified by the buying firm. Through collaboration and interactive coordination mechanisms, the buyer and the offshore suppliers are willing to maximize their supply chain profitability. Finally, the buyer should retain the signed offshore suppliers in its supply base until the contract term expires as far as the supplier does not make any contract violation.

To measure TRC, RPC, and IVC for the *n* number of offshore suppliers in the supply base, we adopt the cost functions developed in the study of Nam et al. (2011) and apply their equation formulas to our model. Let $TC_1(n)$ denote the combined cost of TRC, RPC, and IVC associated with *n* number of suppliers by time *t*. By utilizing the cost formulas of Nam et al. (2011), we assess the combined cost as follows: $TC_1(n) = TRC(n) + RPC(n) + IVC(n) = a^{TC1}n^2 + b^{TC1}n + c^{TC1}$. If the buyer makes a contract with *n* suppliers, we define the total cost per unit time over a procurement period as $TC_1(n)/T$. Regarding the LGC, we define it as the sum of suppliers' outbound transportation cost (TPC) and the buyer's inventory cost (INC). The TPC refers to offshore suppliers' delivery costs to the buyer. We note that we exclude in our model the outbound transportation cost for shipping out finished products to customers from the buyer's warehouse or distribution center to analyze how the total supply chain management cost is affected by the number of offshore suppliers retained in the supply base. If the number of offshore suppliers in the supply base increases, the outbound lot size for each supplier decreases, which may cause increased TPC due to the loss of economies of scale (Su & Levina, 2011). Thus, we assume that the TPC has a functional relationship to the number of offshore suppliers and define it as follows: $TPC(n) = f^{TP}(n) = a^{TP}n^2 + b^{TP}n + c^{TP}$ and $\partial f^{TP}/\partial n > 0$. Like the TPC, the INC is also affected by the number of offshore suppliers in the supply base. Let $\$A_1$ be the cost per order and $\$A_2$ the holding cost per unit. We assume that the buyer contracts with *n* offshore suppliers and orders the same quantity to each supplier. Let $Q_A = D(P)/n$ refer to the order quantity for each supplier and Q_A^* be the economic order quantity for each supplier. The total ordering cost associated with the *n* offshore suppliers, V_o(n), is then given by: $(n)(Q_A/Q_A^*)(\$A_1) = \{D(P)/Q_A^*\}(\$A_1)$. The total holding cost incurred by working with *n* offshore suppliers, $V_h(n)$, is obtained by: $(n)(Q_A^*/2)(\$A_2)$. Hence, the total INC associated with *n* number of offshore suppliers can be expressed as follows: $INC(n) = V_0 + V_h$(n) = $f^{IN}(n) = b^{IN}n + c^{IN}$. Let $TC_2(n)$ refer to the LGC that includes the TPC and the INC associated with *n* number of offshore suppliers in the supply base. Then, we have $TC_2(n) = LGC(n) = f^{TP}(n) + f^{IN}(n) = a^{TP}n^2 + (b^{TP} + b^{IN})n + (c^{TP} + c^{IN})$. Let $a = a^{TC1} + a^{TP}, b = b^{TC1} + b^{TP} + b^{IN}$, and $c = c^{TC1} + c^{TP} + c^{IN}$. In its final analysis, the total supply chain management cost, $TC(n)$, by time *T* can be assessed as follows:

$$\boldsymbol{TC(n) = TC_1(n) + TC_2(n) = an^2 + bn + c} \quad \textbf{(5)}$$

3.2.3 Supply chain profitability when adopting n number of offshore suppliers

Let $P(t) = \alpha(t)S(n)$ be the unit retail price that the buying firm charges end-consumers at time *t*, where $\alpha(t) \in [1, \infty)$ refers to a buyer's profit markup upon the average wholesale price of the *n* offshore suppliers. Let $\beta(t)S(n)$ be the unit production cost, where $\beta(t) \in (0,1]$. We assume that the actual market demand based on the price at time *t* is linear (Weng, 1995; Petruzzi & Dada, 1999) such as $D\{P(t)\} = a - bP(t)$, where a = maximum market size, $a > 0$ and $b > 0$. Let $\pi^B(p, n, s)$ refer to the buying firm's profit before its total supply chain management cost associated with $P(t), n(t)$, and $S(n)$ is subtracted. Since $P(t) = \alpha(t)S(n)$, the buyer's profit at time *t*, $\pi^B(p, n, s)$, can be defined as follows:

$$\pi^B(p, n, s) = D(P)(P - S) = (\alpha - 1)(aS - b\alpha S^2) \quad (6)$$

Let $\pi^S(p,n,s)$ be the total profit of each offshore supplier's profit before its supply chain management cost is subtracted. Then, we have:

$$\pi^S(p,n,s) = D(P)(1-\beta)S = (1-\beta)(aS - b\alpha S^2) \quad (7)$$

Hence, the supply chain profit before the total supply chain management cost is subtracted, $\pi^C(p,n,s)$, and the expected supply chain profit at time t, $\pi^{EC}(p,n,s)$, are respectively expressed as follows:

$$\pi^C(p,n,s) = \pi^B(p,n,s) + \pi^S(p,n,s) = (\alpha-\beta)[aS - b\alpha S^2] \quad (8)$$

$$\pi^{EC}(p,n,s) = E\{\pi^C(p,n,s)\} = (\alpha-\beta)[aE(S) - b\alpha E(S^2)] = (\alpha-\beta)[aM - b\alpha G] \quad (9)$$

3.3 The stochastic optimal control models

Let T be the ending time of the planned sourcing horizon, n_1 the minimum number of suppliers, n_2 the maximum number of suppliers, and $\mathbb{Z}^+$ a non-negative integer. Then, the number of eligible offshore suppliers included in the list is given by: $n(t) = n \in \mathbb{Z}^+$ for all $t \in [0,T]$. To develop the stochastic optimal control model, we relax the conditional properties for $n(t) = n \in \mathbb{Z}^+$. Based on Equations (1) and (8), we derive the stochastic optimal control formulation of supply chain profit maximization by time T as follows:

$$Max\ E\left[\int_0^T \{\pi^C(p,n,s) - TC(n)/T\}dt\right] \quad (10)$$

$$s.t\ \ dS = R_1(n)S\,dt + R_2(n)S\,dW,\ S(0) = S_0 \quad (11)$$

Note that the stochastic optimal control model illustrated above does not provide a closed-form of an optimal solution. Hence, to get a closed-form or suitable solution for our study, we attempt to develop an alternative approach by converting the stochastic optimal control to the optimal control model.

3.3.1 The optimal control models

In this section, withholding the condition of $n(t) \in \mathbb{Z}^+$ and redefining $n(t) \in \mathbb{R}^+$ as a piecewise continuous function of time t, we transform the objective function and constraints of the stochastic optimal control model into functional equations fitting for an optimal control model. We first set the expected supply chain profit as an objective function and then develop constraints on the drift and volatility terms of Equation (11). Since the expected supply chain profit (π^{EC}) is affected by drift and volatility as shown in Equation (9), we treat the two terms as constraints in our optimal control model. We derive the optimal control model for the expected maximum supply chain profit by time T as follows:

$$Max\ \left[\int_0^T \{(\alpha-\beta)(aM - b\alpha G) - \frac{TC}{T}\}dt\right] \quad (12)$$

$$\text{s.t}\quad \dot{M} = R_1 M,\ M(t_0) = S_0 \quad (13)$$

$$\dot{G} = \{2R_1 + R_2^2\}G,\ G(t_0) = S_0^2 \quad (14)$$

$$n_1 \le n(t) \le n_2 \quad (15)$$

Working from Equations (12) to (15), we now develop decision policies and procedures to identify the optimum number of offshore suppliers in the supply base to meet the goals of a supply plan minimizing the risk of supplier uncertainty over the supply planning time horizon. To start developing the optimal solutions, we first provide the Hamiltonian and Lagrangian equations' characterization as follows:

$$H = (\alpha-\beta)(aM - b\alpha G) - TC/T + \lambda_1 R_1 M + \lambda_2\{2R_1 + R_2^2\}G \quad (16)$$

Let $O(n,t) = \partial H/\partial n$, then $H_n = O(n,t) = -(TC_n/T) + (R_{1n})(\lambda_1 M) + 2(R_{1n} + R_2 R_{2n})(\lambda_2 G)$ (17)

From Equation (16), we know that co-states, λ_i for i = 1 and 2, satisfy the following equations:

$$\dot{\lambda_1} = -\partial H/\partial M = -H_M = -\lambda_1 R_1 - a(\alpha-\beta) \text{ with } \lambda_1(T) = 0 \quad (18)$$

$$\dot{\lambda_2} = -\partial H/\partial G = -H_G = -(\lambda_2)(2R_1 + R_2^2) + b\alpha(\alpha-\beta) \text{ with } \lambda_2(T) = 0 \quad (19)$$

Then, we have:

$$\lambda_1(t) = \left[exp\left\{-\int_0^t R_1 dx\right\}\right]\left[\int_t^T a(\alpha-\beta)exp\{\int_0^x R_1 dy\}dx\right] \quad (20)$$

$$\lambda_2(t) = -\left[exp\left\{-\int_0^t (2R_1 + R_2^2)dx\right\}\right]\left[\int_t^T \alpha b(\alpha-\beta)exp\{\int_0^x (2R_1 + R_2^2)dy\}dx\right] \quad (21)$$

The Lagrangian (L) with multipliers (W_1 and W_2) is then formulated as follows:

$$L = H + w_1(n_2 - n) + w_2(n - n_1) \quad (22)$$

Based on the outputs from Equations (18) to (21), two conditions for obtaining an optimal solution are characterized as follows:

$$L_n = \partial L/\partial n = O(n,t) + w_2 - w_1 = 0 \quad (23)$$

$$w_1 \ge 0, w_1(n_2 - n) = 0, w_2 \ge 0, \text{ and } w_2(n - n_1) = 0 \quad (24)$$

Utilizing the properties of Equations (23) and (24), we propose Theorem 1 to identify the optimum number of offshore suppliers in the presence of supplier uncertainty and illustrate how the optimum number should be determined under the conditions stated in Theorem 1.

Theorem 1. For the given S_0, the optimum number of suppliers at time *t*, $n^*(t)$, is as follows:

$$n^*(t) = \begin{cases} n_1 & if\ O(n,t) < 0 \\ \in (n_1, n_2) & if\ O(n,t) = 0 \\ n_2 & if\ O(n,t) > 0 \end{cases}$$

Proof. If $O(n,t) < 0$, then Equation (23) requires $w_1 = 0$. Hence, from Equation (24), we have $n^* = n_1$. If $O(n,t) > 0$, then Equation (23) requires $w_2 = 0$. Thus, from Equation (24), we have $n^* = n_2$. If there exists $n(t)$ such that $O(n,t) = 0$ over an interval of time, it is considered that *n* might be the number existing between n_1 and n_2. When $O(n,t) < 0$ at time *t*, the optimum quantity of offshore suppliers in the supply base at time *t* must be determined by the minimum level, i.e., $\{n(t) = n_1\}$. On the contrary, if $O(n,t) > 0$ at time t, then the optimum number of offshore suppliers at time *t* needs to be determined at the maximum level, i.e., $\{n(t) = n_2\}$.

Theorem 1 implies that the buying firm can determine the optimum number of offshore suppliers based on the sum of the expected marginal profit and the marginal values of associated state variables at time *t*. For example, if the outcome of Equation (17) shows positive, the optimal solution for the buying firm is to obtain the maximum number of offshore suppliers. If it is negative, getting the minimum number of offshore suppliers would be the firm's optimal decision. When it gives zero, the optimal solution lies between the minimum and maximum numbers. If the output of Equation (17) cannot be sustained as zero during an interval of time, a bang-bang control is considered an optimal solution. In this case, the buyer may have to switch the optimum number of offshore suppliers from the maximum to minimum or vice versa depending on the indicated value given by Equation (17) over a planned supply period.

3.3.2 Determining the optimum number of suppliers (n) under a short-term contract scenario*

In this section, we consider a case that the buying firm's strategic decision on the duration of the contract with its suppliers is short-term over its planned sourcing horizon and explore the optimization process for supplier uncertainty mitigation while maximizing supply chain surplus. We assume that the buying firm has *n* eligible offshore suppliers available in its preference list and decide the contract duration when employing them. Over the planned sourcing horizon, the buyer can switch its suppliers, extend the contract period, and add or reduce the number of suppliers whenever it finds better suppliers for its goal achievement. However, the buyer cannot change its signed supplier without completing the contract term. If the contract term starts at t_i and ends at t_j, then it is expressed as follows: $n(t) = n$ for $t \in [t_i, t_j]$ and $i < j$. Let $t_0 = 0$ and $K(n,t) = (R_{1n})\lambda_1 M + 2(R_{1n} + R_2 R_{2n})\lambda_2 G$, then we have $O(n,t) = -(TC_n/T) + K(n,t)$ and $\dot{O}(n) = dK(n,t)/dt = \dot{K}(n) = (R_{1n})\{\dot{\lambda}_1 M + \lambda_1 \dot{M}\} + 2(R_{1n} + R_2 R_{2n})(\dot{\lambda}_2 G + \lambda_2 \dot{G})$.

Proposition 3. For a given $n \in [n_1, n_2]$, $K(n,t) = S_0 a(\alpha-\beta)\left(\frac{R_{1n}}{R_1}\right)\{e^{R_1 T} - e^{R_1 t}\} - 2S_0^2 \alpha b(\alpha-\beta)\left(\frac{R_{1n}+R_2 R_{2n}}{2R_1+R_2^2}\right)\{e^{(2R_1+R_2^2)T} - e^{(2R_1+R_2^2)t}\}$. $K(n,0) = (\alpha-\beta)S_0\left[a\left(\frac{R_{1n}}{R_1}\right)(e^{R_1 T}-1) + 2\alpha b S_0\left\{\frac{2(R_{1n}+R_2R_{2n})}{(2R_1+R_2^2)}\right\}\{e^{(2R_1+R_2^2)T}-1\}\right]$ and $K(n,T) = 0$. $\dot{K}(n,t) = (\alpha-\beta)S_0\left[2b\alpha(R_{1n}+R_2R_{2n})S_0 e^{(2R_1+R_2^2)t} - aR_{1n}e^{R_1 t}\right]$.

Proof.

a) Since $\lambda_1(t) = \left\{\frac{a(\alpha-\beta)}{R_1}\right\}\{e^{R_1(T-t)} - 1\}$ and $\lambda_2(t) = -\left\{\frac{\alpha b(\alpha-\beta)}{(2R_1+R_2^2)}\right\}\{e^{(2R_1+R_2^3)(T-t)} - 1\}$, $K(n,t) = S_0 a(\alpha-\beta)\left(\frac{R_{1n}}{R_1}\right)\{e^{R_1 T} - e^{R_1 t}\} - 2S_0^2 \alpha b(\alpha-\beta)\left(\frac{R_{1n}+R_2R_{2n}}{2R_1+R_2^2}\right)\{e^{(2R_1+R_2^2)T} - e^{(2R_1+R_2^2)t}\}$.

b) $Since\ K(n,0) = K(n,t=0)$ and $K(n,T) = K(n,t=T)$, it follows by (a).

c) Since $\dot{\lambda}_1 = -R_1\lambda_1 - a(\alpha-\beta)$, $\dot{\lambda}_2 = -(2R_1+R_2^2)\lambda_2 + b\alpha(\alpha-\beta)$, $\dot{M} = R_1 M$, and $\dot{G} = b\alpha(\alpha-\beta) - (2R_1+R_2^2)\lambda_2$, then $\dot{K}(n,t) = (\alpha-\beta)S_0\left[2b\alpha(R_{1n}+R_2R_{2n})S_0 e^{(2R_1+R_2^2)t} - aR_{1n}e^{R_1 t}\right]$.

Corollary 1. When $TC_n(n) > 0$ for all $n \in [n_1, n_2]$ and a given S_0,

a) If $K(n_2, 0) > TC_n/T$ and $\dot{K}(n,t) \le 0$, then the optimum number of suppliers at time *t* is determined is as follows:

$$n^*(t) = \begin{cases} n_2 & \text{for } 0 \le t \le t_1 \\ \in (n_1, n_2) & \text{for } t_1 < t < t_2 \\ n_1 & \text{for } t_2 \le t \le T, \end{cases}$$

where, $t_1 = Sup\{0 \le t < T \mid O(n_2, t) > 0\}$ and $t_2 = inf\{t_1 < t \le T \mid O(n_1, t) < 0\}$.

b) If $K(n_2, 0) > TC_n/T$ and the function of $O(n, t)$ is either convex or concave, then $n^*(t) = n_2$ for $0 \le t \le t_1$ and $n^*(t) = n_1$ for $t_1 < t \le T$ where $t_1 = Sup\{0 \le t < T \mid O(n_2, t) > 0\}$.

c) If $K(n_1, 0) < TC_n/T$ and either $\{K(n_1, 0) \ge 0$ and $\dot{K}(n_1, t) \le 0\}$ or $\{K(n_1, 0) \le 0$ and $\dot{K}(n_1, t) \ge 0\}$, then $n^*(t) = n_1$ for $0 \le t \le T$.
d) If $K(n_1, 0) < TC_n/T$ and the function of $O(n, t)$ is convex, then $n^*(t) = n_1$ for $0 \le t \le T$.
e) If $K(n_1, 0) < TC_n/T$ and the function of $O(n, t)$ is concave, then $n^*(t) = n_1$ for $0 \le t \le t_1$, $n^*(t) = n_2$ for $t_1 < t < t_2$, and $n^*(t) = n_1$ for $t_2 \le t \le T$ where, $t_1 = Sup\{0 \le t < T \mid O(n_1, t) < 0\}$ and $t_2 = inf\{t_1 < t \le T \mid O(n_1, t) < 0\}$.

Proof.
a) If $K(n_2, 0)T > TC_n$, then $O(n, 0) > 0$. Since $\lambda_1(T) = \lambda_2(T) = 0$ and $TC_n(n) > 0$, we have $O(n, T) = -\{TC_n(n)/T\} < 0$. If $\dot{K}(n, t) \le 0$ for $t \in [0, T]$, then the function of $O(n, t)$ is non-increasing for all n, which means that there exists at least one occurrence of $n \in (n_1, n_2)$ where $O(n, t) = 0$ for $t \in (0, T)$. This allows us to conclude that $n^*(t) = n_2$ for $t \in [0, t_1]$ where $t_1 = Sup\{0 \le t < T \mid O(n_2, t) > 0\}$, and $n^*(t) = n_1$ for $t \in [t_2, T]$ where $t_2 = Inf\{t_1 < t \le T \mid O(n_1, t) < 0\}$ by Theorem 1.
b) If $K(n_2, 0)T > TC_n$, then $O(n, 0) > 0$ and $O(n, T) < O$. When $O(n, T) < 0 < O(n, 0)$ and the function of $O(n, t)$ is either convex or concave for all n, we can conclude that there exists a t such that $O(n, t) = 0$.

Therefore, $n^*(t) = n_2$ for $t \in [0, t_1]$ where $t_1 = Sup\{0 \le t < T \mid O(n_2, t) > 0\}$, and $n^*(t) = n_1$ for $t_1 < t \le T$ by Theorem 1.
c) If $K(n_1, 0) < TC_n/T$, then $O(n_1, 0) < 0$. When $O(n_1, 0) - O(n_1, T) = K(n_1, 0)$ and if $\dot{K}(n_1, t) \le 0$ and $K(n_1, 0) > 0$ for all *t*, then $O(n_1, t) < 0$ for all *t*. If $\dot{K}(n_1, t) \ge 0$ and $K(n_1, 0) < 0$ for all *t*, then $O(n_1, t) < 0$ for all *t*. Therefore, we have $n^*(t) = n_1$ for $0 \le t \le T$ by Theorem 1.
d) If $K(n_1, 0) < TC_n/T$ and the function of $O(n_1, t)$ is convex for a given n_1, then $n^*(t) = n_1$ for $0 \le t \le T$ by Theorem 1.
e) If $K(n_1, 0) < TC_n/T$ and the function of $O(n_1, t)$is concave for a given n_1, then $n^*(t) = n_1$ for $0 \le t \le t_1$, $n^*(t) = n_2$ for $t_1 < t < t_2$, and $n^*(t) = n_1$ for $t_2 \le t \le T$ where $t_1 = Sup\{0 \le t < T \mid O(n_1, t) > 0\}$ and $t_2 = inf\{t_1 < t \le T \mid O(n_1, t) < 0\}$.

Corollary 2. When $TC_n(n) > 0$ for all $n \in [n_1, n_2]$ and a given S_0,

a) If $K(n_1, 0) < TC_n/T$ and $\dot{K}(n) \ge 0$, then the optimum number of suppliers at time *t* is determined is as follows:

$$n^*(t) = \begin{cases} n_1 & \text{for } 0 \le t \le t_1 \\ \in (n_1, n_2) & \text{for } t_1 < t < t_2 \\ n_2 & \text{for } t_2 \le t \le T, \end{cases}$$

where, $t_1 = Sup\{0 \le t < T \mid O(n_1, t) < 0\}$ and $t_2 = inf\{t_1 < t \le T \mid O(n_2, t) > 0\}$.

b) If $K(n_1, 0) < TC_n/T$ and the function of $O(n)$ is either convex or concave, then $n^*(t) = n_1$ for $0 \le t \le t_1$, and $n^*(t) = n_2$ for $t_1 < t \le T$ where, $t_1 = Sup\{0 \le t < T \mid O(n_1, t) < 0\}$.
c) If $K(n_2, 0) > TC_n/T$ and the function of $O(n_1, t)$ is concave, then $n^*(t) = n_2$ for $0 \le t \le T$.
d) If $K(n_2, 0) > TC_n/T$ and the function of $O(n_2, t)$ is convex, then $n^*(t) = n_2$ for $0 \le t \le t_1$, $n^*(t) = n_1$ for $t_1 < t < t_2$, and $n^*(t) = n_2$ for $t_2 \le t \le T$ where $t_1 = Sup\{0 \le t < T \mid O(n_2, t) > 0\}$ and $t_2 = inf\{t_1 < t \le T \mid O(n_2, t) > 0\}$.
e) If $K(n_1, 0) < TC_n/T$ and either $\{K(n_2, 0) \ge 0$ and $\dot{K}(n_2, t) \le 0\}$ or $\{K(n_2, 0) \le 0$ and $\dot{K}(n_2, t) \le 0\}$, then $n^*(t) = n_2$ for $0 \le t \le T$.

Proof.
a) If $K(n_1, 0) < TC_n/T$, then $O(n, 0) < 0$. Since $TC_n(n) < 0$, $O(n, T) > 0$. If $\dot{K}(n) \ge 0$ for $t \in [0, T]$, then the function of $O(n, t)$ is non-decreasing for all n and there exists at least one occurrence of $n \in (n_1, n_2)$ where $O(n, t) = 0$ for $t \in (0, T)$. Hence, $n^*(t) = n_1$ for $t \in [0, t_1]$ where $t_1 = Sup\{0 \le t < T \mid O(n_1, t) < 0\}$ and $n^*(t) = n_2$ for $t \in [t_2, T]$ where $t_2 = Inf\{t_1 < t \le T \mid O(n_2, t) > 0\}$ by Theorem 1.

b) If $K(n_1, 0) < TC_n/T$, then $O(n_1, 0) < 0$ and $O(n_2, T) > O$. When $O(n_1, 0) < 0 < O(n_2, T)$ and if the function of $O(n)$ is either convex or concave for all n, then there exists a t such that $O(n, t) = 0$. Therefore, $n^*(t) = n_1$ for $t \in [0, t_1]$ where $t_1 = Sup\{0 \leq t < T | O(n_1, t) < 0\}$ and $n^*(t) = n_2$ for $t_1 < t \leq T$ by Theorem 1.
c) If $K(n_2, 0) > TC_n/T$, then $O(n_2, 0) > 0$. When $O(n_2, T) > 0$ and if the function of $O(n)$ is convex, then $O(n_2, t) > 0$ for all *t*. Therefor $n^*(t) = n_2$ for $0 \leq t \leq T$ by Theorem 1.
d) If $K(n_1, 0) > TC_n/T$, then $O(n_2, 0) > 0$. When $O(n_2, T) > 0$ and if the function of $O(n_2, t)$ is concave, then $O(n_2, t) > 0$ for all *t*. Hence $n^*(t) = n_2$ for $0 \leq t \leq T$ by Theorem 1.
e) If $K(n_1, 0) < TC_n/T$, then $O(n_2, 0) > 0$. When $O(n_2, 0) - O(n_2, T) = K(n_2, 0)$ and if $\dot{K}(n) \leq 0$ and $K(n_2, 0) > 0$, then $O(n_2, t) > 0$ for all *t*. If $\dot{K}(n) \geq 0$ and $K(n_2, 0) < 0$, then $O(n_2, t) > 0$ for all *t*. Hence, $n^*(t) = n_2$ for $0 \leq t \leq T$ by Theorem 1.

Corollary 3. For $TC_n = 0$,
a) When $K(n, 0) > 0$ and $\dot{K}(n) \leq 0$, $n^*(t) = n_2$ for $0 \leq t \leq T$.
b) When $K(n, 0) < 0$ and $\dot{K}(n) \geq 0$, $n^*(t) = n_1$ for $0 \leq t \leq T$.

Proof.
a) If $TC_n = 0, K(n, 0) > 0,$ and $\dot{K}(n) \leq 0$, then we have $O(n, 0) > O(n, T) = 0$ and $O(n, t) > 0$ for all $0 \leq t \leq T$. Therefore, $n^*(t) = n_2$ for $0 \leq t \leq T$ by Theorem 1.
b) If $TC_n = 0$, $K(n, 0) < 0,$ and $\dot{K}(n) \geq 0$, then we have $O(n, 0) < O(n, T) = 0$. Hence, $n^*(t) = n_1$ for $0 \leq t \leq T$ by Theorem 1.

Corollaries 1, 2, and 3 illustrate how the buyer can acquire supplies from short-term contracting offshore suppliers in the most timely and cost-effective way during a procurement project time horizon based on the given conditional properties. Under the conditions of (a) in Corollary 1, for example, the buyer should start with the maximum number of offshore suppliers and then gradually reduce the number over the planned supply project horizon. In cases (c) and (d), the buyer should contract with the minimum number of suppliers throughout the procurement cycle. However, under the conditions of (b) and (e) in Corollary 1 and Corollary 2, the optimal decision on the number of short-term contracting suppliers over the procurement project should be bang-bang control in the case that the function of $\{O(n, t) = 0$ for $n \in (n_1, n_2)\}$ cannot be sustained over the procurement project period. Under the conditions of (a) in Corollary 2, the buyer should start with the minimum number of offshore suppliers and then gradually increase the number over the planned supply project horizon. The (c) and (d) of Corollary 2 show that the buyer should work with the maximum number of short-term contract offshore suppliers throughout the procurement cycle. Corollary 3 implies that if the total marginal cost for supply chain management with the chosen number of suppliers (*n*) shows a minimum to even zero, the buyer may need to decide whether to contract with a maximum or a minimum number of suppliers, depending on the conditions of $\dot{K}(n)$.

3.3.3 Determining the optimum number of suppliers (n) under a long-term contract scenario*

As the timeframe of a contract expands, the difficulty in predicting the risk of supply disruption increases. This section examines how to determine the optimum number of suppliers in the context where the firm's strategic decision on the contractual duration with its suppliers is long-term. We assume that the buyer contracts with *n* offshore suppliers and retains them from the beginning to the end of its sourcing plan horizon. In this long-term contract scenario, we consider that $n(t) = n \in \mathbb{Z}^+$, for $t \in [0, T]$. Let $\Phi(n, T) = (\alpha - \beta)[(aS_0/R_1)\{\exp(R_1T) - 1\} - \{b\alpha S_0^2/(2R_1 + R_2^2)\}\{\exp(2R_1T + R_2^2T) - 1\}] - TC$.

Theorem 2. If $n(t) = n \in \mathbb{Z}^+$, for all $t \in [0, T]$, then the optimum number of offshore suppliers, n^*, can be expressed as follows:

$$\begin{aligned} &\text{Max}_{\ n}\ \Phi(n, T) \\ &\text{s.t}\quad n_1 \leq n \leq n_2 \\ &\qquad\ n \in \mathbb{Z}^+ \end{aligned}$$

Proof. Since $n(t) = n \in \mathbb{Z}^+$ for all $t \in [0, T]$, we have $G(t) = S_0^2 \exp\{2R_1(n) + R_2^2(n)\}t$ by Equation (4), $M(t) = S_0 \exp\{R_1(n)t\}$ by Equation (3), and $\pi^{EC}(p, n, s) = (\alpha - \beta)\{aM(t) - b\alpha G(t)\}$ by Equation (9). The expected supply chain profit by time $T, \Phi(n, T)$, can be expressed as follows: $\Phi(n, T) = \int_0^T \{\pi^{EC}(p, n, x) - TC(n)/T\} dt = \int_0^T [(\alpha - \beta)\{aM(t) - b\alpha G(t)\} - TC(n)/T] dt =$
$(\alpha - \beta)[(aS_0/R_1)\{\exp(R_1T) - 1\} - \{b\alpha S_0^2/(2R_1 + R_2^2)\}\{\exp(2R_1T + R_2^2T) - 1\}] - TC.$

Theorem 2 helps us identify the optimum number of long-term offshore suppliers using the nonlinear integer programming approach under supplier uncertainty. It implies that the buyer can determine the optimum number of long-term contract offshore suppliers based on the objective function of supply chain profit maximization with the following conditions: $n(t) = n \in \mathbb{Z}^+$ for all $t \in [0, T]$ and $n_1 \leq n \leq n_2$.

4. Numerical Examples

This section presents some numerical examples to demonstrate how buying firms can use our models when deciding the optimal number of offshore suppliers in their supply base. As we explained in Section 3.1, our optimization model starts with creating a list that includes n number of preferred potential suppliers who can satisfy the buying

firm's evaluation criteria and meet its business needs. Let us assume that that the firm has identified eight preferred suppliers and decided the duration of the contractual relationship with the chosen suppliers to be two years such that $t_0 = 0 \leq t \leq T = 2$ and $n_1 = 2 \leq n(t) \leq n_2 = 8$.

Next, by using the weighted factor scoring method illustrated in Section 3.1, we measure the composite uncertainty risk level for each supplier in the preference list and rank the preferred suppliers according to their total weighted risk scores. Suppose that the buying firm has identified five critical risk factors associated with supplier uncertainty. They are operational contingency (risk factor 1), geopolitical uncertainty (risk factor 2), logistic complexity (risk factor 3), shortage of skilled and raw material resources (risk factor 4), and natural disasters (risk factor 5). Among the five potential triggers for supply disruption, let us assume that the firm has chosen risk factor 1 as its most important criterion and assigned it a weight of 0.4 ($w_1 = 0.4$), 0.3 for factor 2 ($w_2 = 0.3$), 0.15 for factor 3 ($w_3 = 0.15$), 0.1 for factor 4 ($0.15, w_4 = 0.1$), and 0.05 for factor 5 ($w_5 = 0.05$), respectively. Table 1 provides information on risk factor scores, the average weighted scores of the composite supplier uncertainty risk, and preference ranks of the eight suppliers. We assume that the buying firm will determine a supplier with the lowest total weighted risk factor score as the most preferred one.

Supplier i (i=1, …, 8)	Risk Factor j (j=1,2,3,4,5) Weight	Risk Factor Scores					Total Weighed Risk Factor Score r_i	Preferred Supplier Rank 8 = least preferred) 8 = least preferred)
		j1 0.4	j2 0.3	j3 0.15	j4 0.1	j5 0.05		
Supplier 1	r_{1j}	0.90	0.55	0.25	0.23	0.29	0.6000	8
Supplier 2	r_{2j}	0.31	0.95	0.03	0.92	0.47	0.5290	6
Supplier 3	r_{3j}	0.32	0.21	0.23	0.40	0.56	0.2935	2
Supplier 4	r_{4j}	0.03	0.90	0.82	0.75	0.96	0.5280	5
Supplier 5	r_{5j}	0.50	0.80	0.70	0.10	0.10	0.5600	7
Supplier 6	r_{6j}	0.02	0.05	0.35	0.73	0.64	0.1805	1
Supplier 7	r_{7j}	0.82	0.05	0.89	0.03	0.42	0.5005	4
Supplier 8	r_{8j}	0.09	0.74	0.53	0.08	0.74	0.3825	3

Table 1: Total weighted risk score (r_i) and preferred supplier rank

Having the aggregate risk score estimations (r_i) of the eight preferred suppliers presented in Table 1, we now investigate how the average supplier uncertainty level ($\bar{r}^n$) at time t changes as the number of suppliers in the offshore supply base increases or decreases and summarizes the results in Table 2. For example, if the buying firm decides to make a contract with the top three suppliers of the lowest risk rank in the list (i.e., supplier 6, supplier 3, and supplier 8), the average level of supplier uncertainty associated with the chosen three suppliers ($\bar{r}^3$) is estimated as 0.2855. Table 2 provides the average risk level information about a sourcing set configured by the chosen number of offshore suppliers.

n	2	3	4	5	6	7	8
$\bar{r}^n$	0.2370	0.2855	0.3393	0.3770	0.4023	0.4249	0.4468

Table 2: The average level of supplier uncertainty when using *n* number of suppliers ($\bar{r}^n$)

In the second phase of our modeling process, we consider two cases of procurement planning – short-term contracts as many as needed throughout the two-year sourcing project and long-term contracts held for two years for the sourcing project. We then attempt to identify the optimum number (n*) of preferred offshore suppliers in the list that helps build cost-effective resilience while maximizing supply chain profitability for the two cases. To explore the impact of the uncertainty level ($\bar{r}^n$) on the wholesale price per unit and measure the costs and supply chain profitability when operating with the n number of suppliers, we utilize the US lumber commodity spot price and investigate its stochastic time-evolution process. Based on the historical lumber average price data acquired from May 10, 2021, to June 15, 2021 (Figure 1), we draw the draft and the volatility terms of lumber wholesale price that satisfy the geometric Brownian motion (dS=μS dt+σSdW). We estimate the μ and σ parameters by using the

Maximum Likelihood method. Let S_t be the average wholesale price, C_t be the rate of change in the average wholesale price at time t. The market operating days are 252 days per year. We have the following equations for parameter estimations: $\delta t = 1/252$, $m = 25$, $\sum_{t=1}^{25} C_t = \sum_{t=1}^{25}\{(S_{t+1} - S_t)/S_t\} = -0.484119$, $\bar{C} = -0.01862$, $\sum_{i=1}^{25}(C_i - \bar{C})^2 = 0.0287145$, $\mu = \sum_{t=1}^{m}(C_t)/\{(m)(\delta t)\}$, and $\sigma = \sqrt{\sum_{t=1}^{m}(C_t - \bar{C})^2/(m-1)(\delta t)}$. Using these estimated parameters, we now determine the values of the drift and volatility terms as $\mu = -4.87992$ and $\sigma = 0.5490924$, respectively. Hence, we can define the wholesale price per unit of lumber at time t such that $S(t) = (1670.5)e^{[-(5.03067)\, t + (0.5490924)\{W(t) - W(0)\}]}$ by Equation (2). In Figure 1, we illustrate the comparison of the actual closing average wholesale price with the stochastic price estimated by the GBM.

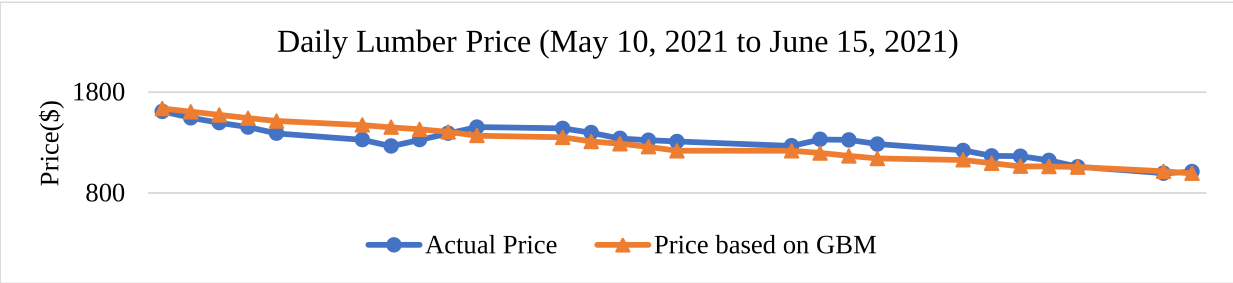

Figure 1: Daily lumber average prices (Nasdaq, 2021)

In our numerical example analysis, we use the historical lumber average price data acquired from June 15, 2019, to June 15, 2021 (Nasdaq, 2021) to estimate μ and σ. The obtained estimation results are summarized as follows: $S(t) = (380.75)e^{[-(0.469006)t + (0.447453)\{W(t) - W(0)\}]}$ where $\mu = 0.569113$, $\sigma = 0.447453$. To quantify the impact of the average wholesale price uncertainty on resilience improvement cost due to the numerical optimization of offshore suppliers, we first convert the equation of $\bar{r}^n(t)$ to the function of n with the regression analysis ($R^2 = 99.81\%$) such that $\{\bar{r}^n \cong a_A + b_A n + c_A n^2\} = -0.00374n^2 + 0.072061n + 0.107054$. As discussed in Section 3.2, we define the drift term (R_1) and the volatility term (R_2) for the average wholesale price per unit of procured items as follows: $R_1(n) = f_1(\mu, \bar{r}^n) = \mu - \theta_1 \bar{r}^n = 0.569113 - (\theta_1)\{\bar{r}^{(n)}\}$ and $R_2(n) = f_2(\sigma, \bar{r}^n) = \sigma + \theta_2 \bar{r}^n = 0.447453 + (\theta_2)\{\bar{r}^{(n)}\}$, where $\theta_1 \in [0, \infty)$ and $\theta_2 \in [0, \infty)$ indicate how much the supplier uncertainty affects the drift and volatility term of the average wholesale price, respectively. We set the parameter values of yearly demand with relevant variables such as that $D(P) = 226800 - 120P$, $\alpha = 1.5, \beta = 0.8$, $S_0 = \$300$, $S_0^2 = \$90{,}000$, and $TC(n) = 1{,}999{,}046 + 18398n + 19354n^2$.

Consider the situation where the buyer chooses an arbitrary number of short-term contracting suppliers in the first year of the project - say the top five offshore suppliers of the lowest risk rank in the list (i.e., suppliers #6, #3, #8, #7, and #4) and then optimize the number of suppliers in the second year with given θ for resilience improvement and maximum supply chain surplus such that $n(t) = 5$ for $0 \le t < 1$ and $n(t) \in [2, 3, 4, 5, 6, 7, 8]$ for $1 \le t \le 2$ under $\theta_1 = 0.4$ and $\theta_2 = 0.1$. Using Equations (3), (4), and (12) to (15), we calculate the expected supply chain profit achieved by using the chosen number of suppliers over the two-year procurement project. Table 3 illustrates seven cases of the total expected supply chain profit obtained based on the conditions such that $n(t) = 5$ for all $t \in [0, 1)$ and $n(t) = n^* = 4$ for all $t \in [1, 2]$. The result of Table 3 indicates that the optimum number of preferred offshore suppliers for the buyer to hold in the second year for the maximum profitability of the project is 4 (i.e., the optimum solution for the second year is to employ the top four suppliers of the lowest risk rank – suppliers #6, #8, #3, and #7).

(5, *n*)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)	(5, 7)	(5, 8)
Profit	$66,420,772	$66,671,141	**$66,750,882**	$66,701,037	$66,552,660	$66,326,162	$66,038,833

*** (5, *n*): $n(t) = 5$ suppliers in the first year and *n* suppliers in the second year: $n(t) \in [2, 3, 4, 5, 6, 7, 8]$**

Table 3: Two-year supply chain expected profit under $\theta_1 = 0.4$ and $\theta_2 = 0.1$

Consider the situation where the buyer decides to optimize the eight offshore suppliers in the list and employ the optimum number of short-term contracting suppliers from the start to the end of the procurement project. Let us define the value of θ_1 as 0.4 and θ_2 as 0.1. In this scenario, we can determine the optimum number of suppliers for the buyer by utilizing the analytical framework developed in Section 3.3. Since $K(n_1, 0) - TC_n/T = -24{,}054{,}120 < 0$, $TC_n = 18398 + 38708n > 0$ for all n and the function of $O(n, t)$ is concave in n for all $t \in [0, T]$, we have $n^*(t) = n_1 = 2$ for $0 \le t \le t_1 = 0.012236$, $n^*(t) = n_2 = 8$ for $0.012236 < t < 1.974675$, and $n^*(t) = n_1 = 2$ for $t_2 = 1.974675 \le t \le 2$ by Corollary 1 (e). The optimal sourcing solution for the maximum supply chain profitability with resilience in this scenario is to hold the offshore suppliers of the lowest disruption probability; two offshore suppliers of the lowest and the second-lowest risk (i.e., supplier #6 and #3) throughout the procurement project period.

Let us now consider the sourcing strategy involving a long-term contact policy to meet the two-year procurement project. That is, once the preferred offshore suppliers are identified and contracted, the buyer must keep them until the contractual relationship expires. For this scenario, we first evaluate how $\bar{r}^n(t)$ can influence the wholesale price of long-term contracting offshore suppliers and quantify the cost and supply chain profitability based on two different sets of price drift and volatility terms. One contains a fixed drift term with varying volatility terms such that θ_1 = 0.4 and θ_2 = 0.05, 0.1, 0.2, 0.4, 0.5, 0.6, 0.8, and 1.2. Table 4 summarizes the expected supply chain profits, and Table 5 shows the optimum number of offshore suppliers evaluated by applying the defined parameters above. The other holds a fixed volatility term with different drift terms such that θ_2 = 0.2 and θ_1 = 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6. 0.7, and 0.8. Based on these parameters, Table 6 provides the optimal number of long-term contracting suppliers and the expected profits obtained from the chosen number of suppliers.

n	θ_2 =0.05	θ_2 =0.1	θ_2 =0.2	θ_2 =0.4	θ_2 =0.5	θ_2 =0.6	θ_2 =0.8	θ_2 =1.2
2	$67,094,813	$65,746,601	$62,873,092	$56,349,365	$52,655,058	$48,638,855	$39,522,199	$15,911,782
3	$67,863,190	$66,293,528	**$62,900,361**	$54,975,762	$50,362,838	$45,252,129	$33,302,654	$312,911
4	$68,331,734	$66,591,982	$62,785,315	$53,676,180	$48,246,615	$42,131,160	$27,453,945	-$15,480,845
5	$68,569,761	$66,701,037	$62,570,672	$52,483,635	$46,350,496	$39,345,317	$22,151,140	-$30,766,701
6	$68,629,146	$66,664,887	$62,288,022	$51,422,099	$44,708,468	$36,952,327	$17,559,543	-$44,721,207
7	$68,547,897	$66,515,907	$61,960,265	$50,508,731	$43,346,369	$34,999,206	$13,828,285	-$56,468,663
8	$68,352,727	$66,276,844	$61,603,352	$49,755,388	$42,283,061	$33,522,271	$11,081,862	-$65,183,560

Table 4: The expected supply chain profit under θ_1=0.4 and θ_2

θ_2	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
n^*	6	5	3	2	2	2	2	2	2

Table 5: The optimal number of long-term contracting offshore suppliers under θ_1=0.4 and θ_2

θ_1	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
n^*	2	2	2	2	3	3	3	3	3

Table 6: The optimal number of long-term contracting offshore suppliers under θ_2=0.2 and θ_1

The characteristic of n in our numerical examples for the long-term contract is based on an integer ($n \in \mathbb{Z}^+$). If we allow n to be a real number ($n \in \mathbb{R}^+$), we may have different outputs. For example, for the given conditions of θ_1 = 0.4 and θ_2 = 0.1, then the optimum number of suppliers is 5.2 (rather than 5), as shown in Table 7. Since the conditional property of $n^*(t)$ is that n must be a positive integer ($n \in Z^+$) throughout the two-year procurement period, we cannot apply Theorem 1 to the numerical optimization process. To address this issue, we convert the stochastic optimal control model to an ordinary optimal control framework and propose Theorem 2, a unique approach that enables us to obtain the closed-form of an optimal solution. For example, for a given set of (θ_1 = 0.4) and (θ_2 = 0.1), the objective function is to Maximize $(\alpha - \beta)[(aS_0/R_1)\{\exp(R_1T) - 1\} - \{\exp(2R_1T + R_2^2T) - 1\}]$ $-TC$ with two constraints that $2 \leq n \leq 8$ and n is an integer. The integer nonlinear programming analysis finds that the objective function maximizes when n=3 (suppliers #6, #3, and #8), and the total supply chain profit is estimated as $62,900,361, which is consistent with the outcome shown in Table 4. This result confirms that the integer nonlinear integer programming approach presented in our model could be used to determine the optimum number of long-term contracting offshore suppliers that can promote both resilience and supply chain profit maximization.

n	5	5.1	5.2	5.3	5.4	5.5
profit	$66,701,037	$66,703,337	**$66,704,233**	$66,703,759	$66,701,951	$66,698,841

Table 7: The expected supply chain profit under θ_1=0.4 and θ_2=0.1

5. Conclusion

As the COVID-19 has exposed the shortcomings of globalized supply chains and logistics networks, the numerical optimization of an offshore supply base with lower-risk suppliers is receiving significant attention from buying firms to consider as a practical option for supply base resilience and risk mitigation. Firms may improve their resilience performance by reducing their foreign supplier dependency and diversifying suppliers in number. However, reshaping their supply base with the increased foreign supplier redundancy may not be equally effective to every firm, and resilience improvement may not always result in greater economic returns. Thus, it is required for supply chain managers and planners to conduct a systematic analysis to ensure whether the chosen number of suppliers presents the optimum condition for the cost-effective resilience level they need to minimize supply disruption risks. To address this issue, we consider procurement firms competing based on price or supply chain efficiency and propose a model illustrating how they should embed supply risk management into their optimal offshore sourcing decisions in search of post-pandemic resilience.

We first demonstrate the creation of a risk-level-based procurement list that covers the entire lifecycle of acquiring the materials or components a firm needs to operate – a strategic process to initially rationalize potential offshore suppliers into a manageable size (*n*) with each ranked in the supplier-specific risk level (r_i). The risk assessment activity of this process is crucial since it sets the phase for planners and managers to estimate the vulnerability of their organization and consider resilience improvement directions to avoid or mitigate future supply disruption risks. Once the risk levels (r_i) of the preferred suppliers in the list with the selection priority are determined, we provide a quantitative modeling framework for them to analyze how to cost-effectively secure the offshore supply base against disruptive risk by obtaining the optimal number of offshore suppliers (n^*). By focusing on the simultaneous impact of wholesale price uncertainty and the number of suppliers held in an offshore supply chain, we provide essential conditional properties of the resulting optimization functions with decision rules to determine the best supplier choice set that can generate resilience for risk mitigation while maximizing supply chain profitability. Taking into consideration the potential cost-saving advantage of multiple supplier competition with price bidding negotiation, we illustrate the tradeoff assessment to clarify how many foreign sources are too resilient with no cost-effectiveness and how few suppliers cause poor resilience with significantly increased disruption risk. We further show how buying firms can make optimum-number-of-offshore-supplier decisions with supply base resilience in different supplier preference ratings under short- and long-term contract procuring situations. For example, we provide the stochastic-optimal-control-based formulations with conditional rules for buyers to analyze how they can determine the optimal number of short-term contract offshore suppliers in the most timely and cost-effective manner over the procurement time horizon. In the long-term contract procurement case where buyers must keep their suppliers until the contract term is over, we develop an integer nonlinear programming model to obtain the closed-form of an optimal solution - a positive integer ($n \in \mathrm{Z}^+$). By converting the stochastic optimal control model to the integer nonlinear programming analysis framework, buyers can configure a supply base with the optimum number of long-term contracting offshore suppliers to meet their sourcing and procurement plan. The findings of our numerical example analysis confirm that the converted optimal control approach could be used to identify the optimum number of long-term contracting suppliers for risk mitigation. The global business environment constantly changes, creating new risks and new opportunities. Thus, the optimal size and mix of offshore suppliers may need to be reshaped periodically. The capability of our model can help global procuring firms continuously reassess their vulnerabilities and determine when they should strategically modify their established optimal supplier quantity to minimize future supply disruptions that inevitably arise.

Our research has some limitations. We mainly focused on the supply-side uncertainty. We assumed that the demand function of the proposed optimization process was deterministic and defined the number of offshore suppliers (*n*) at a given time as the piecewise-continuous function. Our model did not capture the demand-supply coordination risk in evaluating offshore supply partners. It would be valuable to extend the proposed model by incorporating the non-deterministic demand function into the numerical optimization process and examine how the demand-supply coordination risk will affect optimal offshore sourcing decisions. As for the risk-assessment-based optimization, the proposed model considered tier 1 suppliers of buying firms and overlooked the overstock and understock inventory cost when estimating the profit function of the offshore supply chain. In the context of long-distance sourcing under supplier uncertainty, understanding how the supply disruption risk associated with their secondary suppliers and subcontractors can affect resilience investment costs is also worthy of further investigation.

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