**IPRPD**
International Journal of Business & Management Studies
ISSN 2694-1430 (Print), 2694-1449 (Online)
Volume 02; Issue no 07: July, 2021

# Risk-Adjusted Breadth in Active Portfolio Management

**Plamen Patev,[1] Kaloyan Petkov [2]**

[1] *Associate Professor, International College, I Shou University, Kaohsiung, Taiwan*

[2] *Ph.D, Researcher & Business Intelligence Analyst, ABIR Analytics*

## Abstract

*Active portfolio management postulates that investors should seek increasing the breadth of their portfolio. Usually, investors associate breadth with the number of assets. Because of the unknown correlations between models' forecasts and errors, breadth almost always is significantly less than number of assets in investors' portfolio. In order to increase breadth, long investors must increase to higher extend the number of assets in their portfolio leading to ambivalent influence over active risk. One part of the active risk - tracking error can be diversified by increasing the number of assets but with diminishing result. The second part of the active risk - strategy risk constantly increases with number of assets. Therefore, theoretically there must be some optimum level of number of assets in portfolio which maximize alpha and IR of the investment. This optimum will depend on the specifics of investors' benchmark. Taiwanese market with its broad index provokes active investors to increase the breadth significantly. We found for this stock market that breadth increasing can be effective for IR maximization when portfolio involves 15-25 number of assets (or 5%) of the assets. Further increasing in numbers will cause either alpha eating or total active risk increasing and eventually will result in lower IR.*

**Keywords:** Diversification, Breadth, Strategy risk, Alpha correlation

## Introduction

Active management attracts the interest of portfolio managers with its simplicity and rationality. The main idea can be explained in remarkably simple logic: If we know that an asset will provide excessive to its consensus return (according to the investor's benchmark) then why don't we bet on it? These excessive returns are known as active return or alpha ($\alpha$). Investors should invest long in assets with positive expected alpha and short in those with negative alpha. Therefore, everything depends on the forecasting skills of portfolio managers to find which asset will provide excessive to their consensus return. The skills are measured as correlation between forecasted and real return and are described as Information Coefficient (IC). The extent on which these skills are applied for making investment decisions (bets) is known as Breadth (Br). In order to create alpha investors with some level of IC should increase the breadth. Therefore, the breadth is with crucial importance for the investors to achieve the targeted alpha and they must know which the optimum Br is for them. Active management in its original form postulates that investors can control the breadth of their portfolio by increasing number of bets, which means to increase either number of analyzed assets or to increase the frequency of investment. The reason is that if IC can be applied for bigger breadth this will allow the investors better to benefit from forecasting power of the models due to assumption for diagonal correlation matrix which allows to diversify away the active risk. The more asset in portfolio involved the higher alpha opportunity for investors is. However, when the assumption for diagonal matrix is removed and correlation between forecast is taken into account selecting portfolio breadth becomes essential for active portfolio management. Increasing in number of assets will cause additional risk – active risk. The higher risk will lead to decreasing in information ratio (IR) - ratio between active return (alpha) and active risk. Therefore, mechanism behind relation between number of assets, breadth, active risk and IR should be known for investors in order to construct their active portfolio. Here we analyze how the process of increasing number of assets influences breadth and how the breadth impacts active risk. The results of this analysis are tested on Taiwanese stock market.

## Literature review

Without a doubt, the fathers of active management are Grinold and Kahn (G&K) (2000). They first develop this simple idea into a broad and structured theory: active return $\alpha$ can be achieve with assets with high tracking error $\sigma^2$ (high deviation of return from the benchmark portfolio), which are highly scored by investors $z_i$, with high forecasting ability $IC$, .This idea can be described as, $\alpha_i = \sigma_i * IC * z_i$. When this logic is applied for multiple assets and when the scores of these assets are properly standardized[1] it leads to the final variant of so called the Fundamental Law of Active Management (FLAM) has been derived as (1)

**(1)** $$\boldsymbol{IR = IC * \sqrt{Br}}$$

where:

IR is the portfolio's information ratio and $IR = \frac{\alpha}{\sigma}$;

IC - the information coefficient (forecasting power) of the investment strategy;

$Br$ – the breadth of the portfolio measured as number of bets per year, i.e., number of assets and frequency of investment decisions taken over them;

$\alpha$ - active return;

$\sigma$ – active risk.

FLAM and (1) earns popularity as simple yet powerful explanatory tool of investment process. In fact, IR is a ratio between active excessive return and additional risk taken by active investors. It not only presents the value added by active management to the investor's portfolio but describes importance of the breadth. According to (1), higher IR can be achieved either by high IC or high Br. If investors have constant IC (i.e., some fixed skills to predict future excessive return), the only way to increase benefits from these skills is to increase the number of bets, i.e., to apply their forecasting skill to as many as possible assets and as frequently as possible. In other words, investor should increase breadth (Br) or, if the number of investment bets are limited, then the only way to increase results from their investment is to apply managers with better forecasting skills. This is strategy of increasing IC.

FLAM presents the simple but rational idea of the active management. It explains the relations which are easily acceptable and applicable by every investor. With all these pragmatic characteristics though, the initial form of FLAM is based on several assumptions which create problems with its practical interpretation.

*The first problem in (1)* is the assumption that active risk can be described with tracking error - $\sigma^2$. According to the model, the only source of additional active risk[2] is the tracking error of assets. In the original form of FLAM, the active risk is only presented by the tracking errors of the factor model used for alpha forecasting. G&K work with a diagonal correlation matrix of the active risk which additionally leads to assumption that the single source of active risk can be only tracking error. However, this assumption is too straightforward for investors. Obviously, the correlation matrix in most often applied factor models is never diagonal - there is some correlation between residuals of the stocks as they are exposed to risks which are not involved in the main regression model.

*The second problem* with *(1)* is a consequence of the first - the assumption of constant time series IC. The original FLAM uses indicative assumption about constant IC of each active investor. Theoretical definition of IC is correlation between forecasted and realized alpha. In this perspective it presents subjective preciseness of the ability of each manager to forecast future alpha. And because it is difficult to construct time series data for these subjective abilities, it is acceptable to assume some approximate value for manager's IC and to fix it as constant over time. It is easy to be proven that the manager has relatively fixed skills over the time. However, active managers instead of their subjective skills mostly apply factor models for establishing future alpha. In this case each factor model would have different explanatory power during the time. Change of the explanatory power of the models among the periods actually changes the correlation between forecasted and realized values of alphas. This leads to the fact that i) this correlation could be observed and ii) it variates during the time.

*The third problem* in (1) is with explanation what in fact the breadth is. The interpretation of Br is as being indicator for the scope of active management – to which extent or how often the manager's skills (IC) are applied in variable environment ($\sigma^2$). In FLAM's definition it is assumed that breadth is the number of independent forecasts which the manager can undertake. In order to achieve better results from active management, the investors should either increase the number of assets in the portfolio or to apply more often (per unit period) their forecasting skills. If we assume that all investors work with equal frequency of investment[3] the breadth can be reduced to the number of assets in the portfolio over which *independently* the forecasting skills are applied. Apparently, independence is the most restrictive requirement. If investors apply forecasting models, there will be dependance either between models because they use correlated factors or between errors in the models because no model can fully describe the variation in alphas.

Buckle (2004) applies different approach to FLAM - by placing special emphasis on breadth. The new generalized form of the law developed by him demonstrates the importance of alpha correlation for the breadth in

[1] When scores $z_i$ are standardized with average 0 and $\sigma_z = 1$, then $\sum z_i = Br$

[2] if managing portfolio beta over time (known as the market timing) is excluded. G&K explain separately influence of beta changes and regard it as an additional type of active portfolio management. In this paper we also exclude from consideration and leave it as object for other type of research.

3 The frequency of the trading is interesting issue but should be analyzed separately as it relates to different feature of investment process - timing. Throughout all paper we assume that Br is connected only with the number of assets in investor's portfolio.

active management. As a result, we have a more comprehensive understanding of breadth. He proves that breadth can be generally described with (2)

$$\textbf{(2)} \qquad Br = \sum_{i,j} \pi_{ij} \rho^{*}_{ij},$$

where:

$\pi_{ij}$ is the correlation matrix between forecast of i-asset and j-asset;

$\rho^{*}_{ij}$ - inverse forecasts error correlation matrix ($\rho_{i,j}$- correlation matrix of forecasting errors)

The insight of (2) can clearly be explained with a case when investors apply one single-index forecasting model for all analyzed assets (i.e. single forecast applied for multiple assets). In this case, the breadth will be equal to $\rho^{*}_{ij}$ - the correlation in forecasting model's error. Therefore, if this correlation is positive, the breadth could be between 1 and number of assets in portfolio ($1 \leq Br \leq n$). This conclusion is fully consistent with the investment logic. Under conditions of lack of any correlation in errors investors apply their forecasting skills over the all number of assets, i.e. $Br = n$. If the correlation in errors is perfect, then $Br = 1$. Perfectly positive correlation would mean that the investors do not have possibility to apply their forecasting skills to many assets. In fact, they would invest only in one asset as all others are correlated. If the correlation is positive, then even when investors try to apply diversification by increasing the number of assets in their portfolios, they cannot fully diversify the risk. In the presence of any correlation breadth will always be less than the number of assets, i.e. $Br < n$.

Of course, in order to achieve better results, investors apply multiple forecasts. Buckle presents several cases of (2) application for multiple forecasts. As some of the cases are extreme and have only theoretical importance, for us two of the cases are interesting. The first case is in existence of independent forecasts and uncorrelated errors in the models. In fact, this is exactly the case of standard FLAM because of its two assumptions - for independence in forecasts and for a diagonal covariance matrix. Only in this case (2) will always result in $r = n$ , breadth equal to number of assets in portfolio. However, in the investment world we rarely observe lack of correlation of errors in factor models and it is rarely possible to make independent forecasts. Multiple forecasts are typically made by the same risk analysts, or with the same models, or with factors which are not independent, and this leads to dependency in the forecasts. From other side, errors are usually correlated because no forecasting model can fully describe all set of factors influencing alpha.

The most realistic case is when both forecasts and errors are dependent. In these conditions, the key for investors is how this information of dependent forecasts and errors is used. If investors use true error correlation matrix as a risk model in portfolio optimization, then according to Buckle (2004, p.399), (2) is transferring to $Br = \frac{n}{1-\rho} * \frac{1+\rho[n-2-\pi(n-1)]}{1+\rho(n-1)}$ . It explains that investors can achieve breadth extended over the number of assets. In theory, if real correlation matrix $\rho_{i,j}$ is known and some of them are negative, then portfolio optimizers will use them to maximize breadth over number of assets by applying financial engineering. Therefore, knowing the true error correlation $\rho_{i,j}$ would significantly increase the breadth. But in reality, true error correlation cannot be known in advance. In fact, investors cannot use real error correlations but only forecasted in their portfolio optimization models and by this way they sacrifice a lot of breadth. As most investors use very simplified portfolio construction methods it is reasonable to assume that this level of breadth cannot be achieved.

This is serious implication because it drastically decreases breadth of active portfolios. Reason for this is the inability to use information about forecasting errors. The difference between two approaches is on Figure 1:

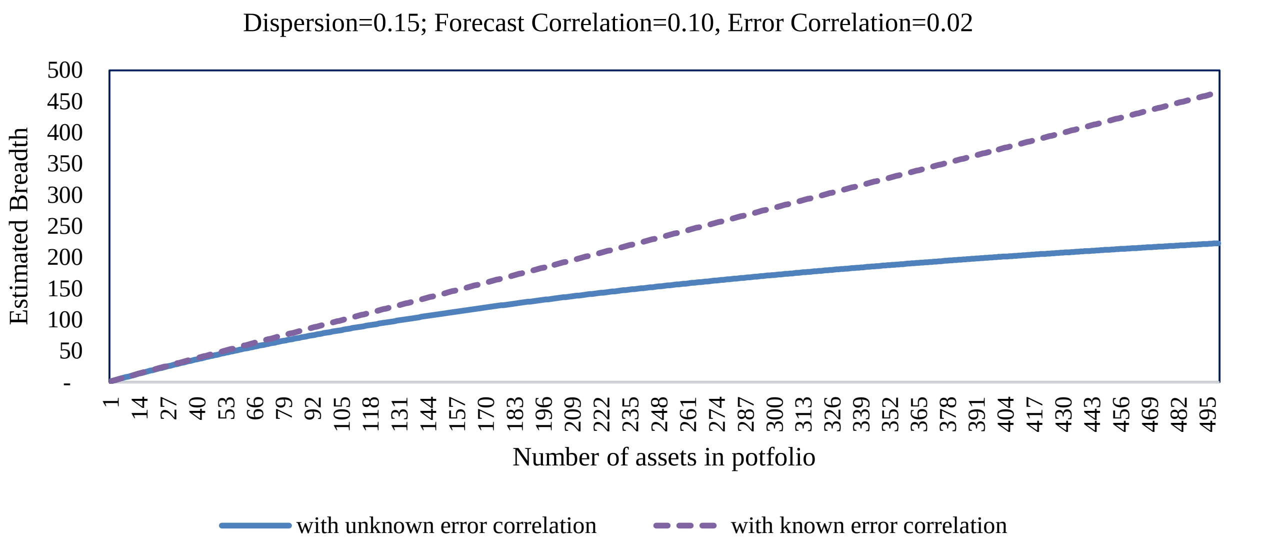


**Figure 1. Estimated Breadth according to Buckle (2004)**

As Buckle (2004, p.399) suggests, because the true error correlation is unknown for the investors, a simplified variant of the formula should be used (3).

$$\mathbf{Br} = \frac{\boldsymbol{n}}{\mathbf{1}+\boldsymbol{\pi}*\boldsymbol{\rho}*(\boldsymbol{n}-\mathbf{1})} \quad \textbf{(3)}$$

where:

$\pi$ is the correlation in forecasts;

$\rho$ – the correlation in errors of forecasts;

From equation (3) can be clearly seen that the breadth is a construct of both alpha and error correlations. This conclusion significantly changes the perception of breadth. Obviously, the relationship between the number of assets in the portfolio and breadth is ambiguous. The influence of the correlation of the errors in the risk model and the correlation between the forecasts leads to situations in which almost always the number of assets differs from breadth, $Br \neq n$ . And in situation with unknown correlation in errors the breadth always be less than number of assets, $Br < n$. From (1) we know that breadth represents the scope of active management. Therefore, it is important for investors to know what number of assets in their portfolio can provide the desired IR. By increasing the number of assets, they may mistakenly assume that they increase the breadth of their investment. When investors apply models with some IC, they try to increase the number of assets hoping to achieve high IR. However, from (3) is clear that breadth can be associated with correlation in forecasts and in errors of the forecasts. It turns out that: if there is such correlation, increasing the numbers of assets does not increase the breadth with the same level. This is significant change in paradigm of the breadth and gives new direction of the development of active management. Initial understanding is that breadth equals number of assets due to association of independent bets, and increasing breadth impacts positively active portfolios by diversifying active risk and thus increasing risk-adjusted performance (IR). However, this paradigm is not realistic in real-world environment where there always are correlations between forecasts and errors. In reality, investors can try to increase breadth by adding more assets, but result will depend on current market conditions like correlation, strategy risk etc. Later Sneddon, L. (2020) confirms the negative impact of return correlations, but shows conflicting evidences in the impact of forecast correlation. Some studies like Heinrich, Shivarova·& Zurek (2021) claim that it is better to build portfolio without diversification traits, which direct disregard for the role of breadth.

Additionally, there is a cost for adding more assets in active portfolio – decreasing alpha. The problem with the breadth and diversification is influencing alpha investing. The normal logic of active investment presented in (1) is to invest in assets with high IR. This could be done by investing in assets with high alpha. That is why the investors prefer to involve such assets in their portfolio. However, increasing breadth requires faster increasing in n. This cause investors to include more and more undesirable assets with less alpha and as a result portfolio alpha will be reduced - a process which can be described as alpha eating. As a result, it is crucially important investors to specify precisely which number of assets in their portfolio will maximize their breadth.

Next direction of the active management development is elaboration on the essence of active risk. Both models (1) and (3) accept that active risk is only as tracking errors of the assets. This interpretation of the risk is considerably basic. FLAM assumes that active risk can be described by the tracking error of the assets and IC is constant. However, later (see Qian&Hua (2004), Ye (2008) etc.) was proven that IC can be very volatile and its standard deviation $\sigma_{IC}$ can influence significantly the total active risk. Forecasting power of the models varies during the time and adds additional uncertainty for forecasted alpha. The complete form of active risk involving $\sigma_{IC}$ is developed by Ding&Martin (2017). They managed to develop a real redux of the FLAM. The full form of active risk is explained by (4)

$$\boldsymbol{\sigma_p^2} = \boldsymbol{\sigma^2} * \left(\boldsymbol{\sigma_{IC}^2} * \boldsymbol{n} + \boldsymbol{\sigma_\epsilon^2}\right), \quad \textbf{(4)}$$

Where

$\sigma_{IC}^2$ the time-series variation of IC;

$\sigma_\epsilon^2$- dispersion of forecasting errors;

In (4) clearly can be seen two different sources of active risk. First source is tracking error $\sigma^2$ - the same as in FLAM. It is unconditional volatility of alpha. In the same manner Ding&Martin (2017) assume diagonal risk matrix for it. However, the tracking error in (4) is multiplied by the second part – strategy risk in form of $(\sigma_{IC}^2 * n + \sigma_\epsilon^2)$. The strategy risk is that part of the total active risk, which is conditioned on the selected investment strategy. Choosing active strategy, i.e., forecasting model investors must take into account the volatility of predicting power of the model (IC).

Introducing the concept of strategy risk is one significant improvement in explanation of active risk. Involving the strategy risk describes more precisely sources of risk in active management. However, the assumption of the breadth as number of assets is still valid in (4) concept of strategy risk. In such way Bucke's model with its correlations is not being considered in risk explanation. (4) also assume diagonal matrix of tracking error. Therefore, the issue about the influence of the breadth over the total active risk remain still not fully revealed in Ding&Martin's framework.

We are going is to involve strategy risk in Buckle's paradigm of breadth. We analyze the influence of strategy risk in the concept of breadth and how this new form of risk changes the essence of the breadth.

## Concept/ Methodology

In existence of correlation between forecasts and errors in these forecasts the scope of active investment will be always differentiated from the number of assets in portfolio. Therefore, to observe the influence of the breadth over the active risk we have to substitute number of assets in (4) with breadth.

The main outcome of (4) is that active risk is associated with tracking error corrected with strategy risk multiplier. We can further modify (4) in order to observe the relationship between Br and n. First part in (4) is the body of active risk which is the unconditional portfolio tracking error ($\sigma^2$) or volatility of alphas. If historical alphas (residual returns) are estimated by CAPM model, then this is equal to the idiosyncratic risk. We apply Gorman, Sapra&Weigand (2010) approach for presenting time series tracking error into cross section realm. They show that idiosyncratic risk of any market model can be expressed in terms of return dispersion (5):

$$\textbf{(5)} \qquad \boldsymbol{\sigma^2 = \sigma^2_{CS}\left[\frac{1}{n} - \frac{1}{N}\right]}$$

where:

N is number of asset in asset universe (benchmark);

$\sigma^2_{CS}$ – alpha dispersion.

Equation (5) allows us to relate the volatility of alphas to the number of assets in the portfolio (n). However, because of the existence of correlation between forecasts and between errors the "real" number of assets in the portfolio is practically never equal to n. Investors usually follow a scenario where investor makes multiple forecasts that are correlated and use in their model factors which hardly could be independent. This resembles Buckle's case with dependent forecasts that dependent errors. In this scenario as it stated above, the breadth will be always less than the number of assets ($Br < n$). Therefore, existence of n in (5) is inappropriate and we have to substitute it with Br.

Second part of the active risk in (4) is strategy risk involves with its multiplier ($\sigma^2_{IC} * n + \sigma^2_{\epsilon}$). It presents the volatility of IC but influenced by two additional elements $\sigma^2_{IC} * n$ and $\sigma^2_{\epsilon}$. The last one is dispersion of the errors in forecasting model - $\sigma^2_{\epsilon}$ which is additive to $\sigma^2_{IC}$. For this additive element we show [see Patev&Petkov (2018)] that the variance of errors from active models can be broken down as a function of IC and cross-section dispersion (6):

$$\textbf{(6)} \qquad \boldsymbol{\sigma^2_{\epsilon} = (1 - IC^2) * \sigma^2_{CS} \approx \sigma^2_{CS}}$$

Since traditional $IC^2$ is close to 0 then the variance of errors can be expressed as dispersion of alphas $\sigma^2_{\epsilon} \approx \sigma^2_{CS}$. Note that if we have higher IC then this becomes less true but in practice ICs over 0.20 or even 0.15 rarely can be observed, which roughly correlates to 70% success rate[4] (70% of investor's forecasts are correct).

Next element in strategy risk in (4) is $\sigma^2_{IC} * n$ where existence of the number of assets n is important. It allows us to observe the cumulative impact of number of assets on the variance of IC. Here n again presents the scope to which the forecasting skills are applied. However, because of (3) we know that the scope of application of IC is differentiated from n. Therefore, n should be substitute with breadth.

Considering this simplification, we can now rewrite equation (4) into (7):

$$\textbf{(7)} \qquad \boldsymbol{\sigma^2_{r_i} = \sigma^2_{CS}\left[\frac{1}{Br} - \frac{1}{N}\right] * \left(\sigma^2_{IC} * Br + \sigma^2_{CS}\right)}$$

(7) states that the active risk is a function of Br, sigma IC and dispersion. The influence of breadth can be presented in both components of the risk – tracking error and strategy risk. Clearly, (7) shows that breadth has opposite impact upon these two components.

***First component of active risk: Tracking error $\boldsymbol{\sigma^2_{CS}\left[\frac{1}{Br} - \frac{1}{N}\right]}$.***

Tracking error is a risk that is independent from the active investment strategy. Investors try to manage this risk by selecting number of assets included in the portfolio Br. It is easy to notice in (5) that by increasing the number of assets in portfolio and approaching N, the tracking error should approach 0. Of course, that is well-known effect of diversification, and many investors increase n in order to diversify away this part of the active risk. By increasing n

[4] Success ratio is another measure of investor's forecasting accuracy that tells what percentage of forecasts will be correct. It is related to the IC through: $Success\ Ratio = \frac{1+IC}{2}$. Therefore IC of 0.10 translates into 55% correct forecasts.

the investor increases the breadth leading to diversification. However, diversification will totally diversify away all tracking risk only if there is no correlation between models (forecasts) and models' errors. Investors in most of the cases follow a scenario where investor makes multiple forecasts that are correlated.

It is easy to see that if there is correlation between forecasts or model's errors then the breadth will be lower than the number of assets. Inputting this equation into the estimation of total active risk over growing number of assets in a universe of 500 stocks we get Figure 2:

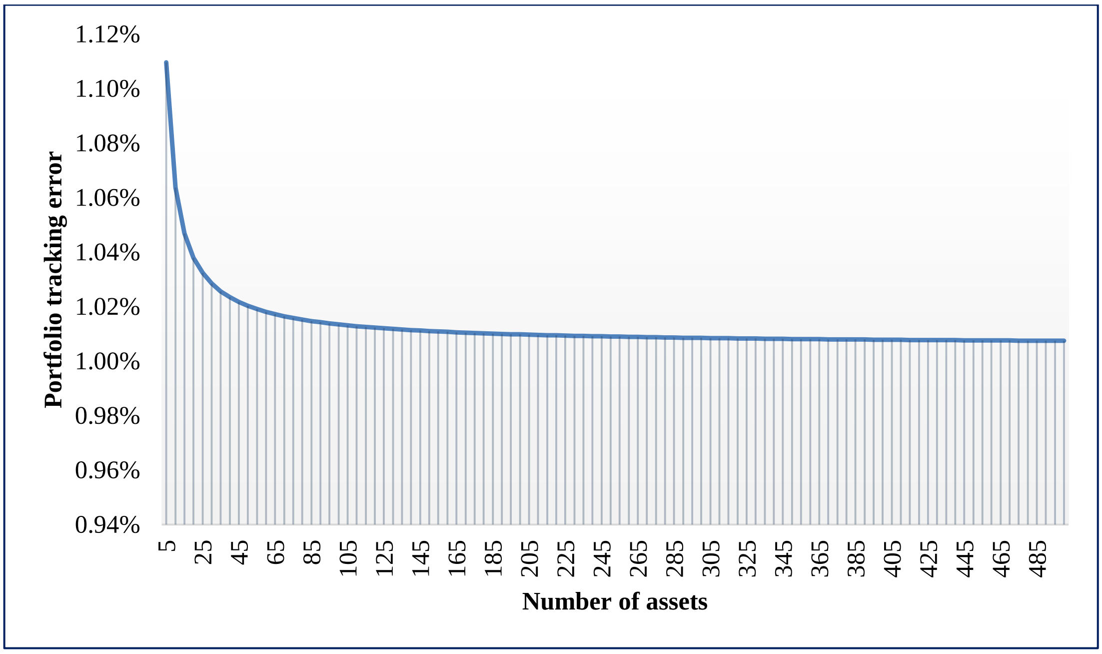


**Figure 2. Total active risk across different portfolio sizes (number of assets)**

This result is crucially important as it suggests that active risk can be only marginally diversified. Diversification benefit can be defined as the benefit to IR when investor increases number of assets held. It is important to stress IR benefit since the goal of active investors is to achieve risk-adjusted alpha and therefore diversification translates into reducing risk. As our simulation suggests, there is no reason to add assets after 20-30 because the diversification benefit gets extremely low. To get better idea how breadth impacts the risk it is required to separately examine the tracking error and strategy risk.

***Second component of the active risk is strategy risk multiplier*** $\left(\sigma_{IC}^2 * Br + \sigma_{CS}^2\right)$.

The strategy risk is that part of the total active risk, which is conditioned on the chosen investment strategy. If the investors apply single- or multiple-index models, variance of IC, $\sigma_{IC}^2$ and the dispersion $\sigma_{CS}^2$ are intrinsic for the model. It is a matter of investors' strategy which exactly forecasting index model to be applied but once selected variation of the model's IC and resultative cross section dispersion are independent from the number of assets and therefore, unmanageable with changing the breadth. Therefore, we assume that $\sigma_{IC}^2$ and $\sigma_{CS}^2$ are constant for the investors. What could be controlled is the scope of selected model - the number of assets the model will be applied on. Responding to satisfactory level of model's IC, investors may choose to increase number of assets to benefit from that IC. However, forecasting power of every model is not constant throughout the time because there is $\sigma_{IC}^2$. According to (7) if n increases, variation of IC will be multiplied with higher number ($\sigma_{IC}^2 * Br$) resulting in higher strategy risk multiplier. The logic behind this relation is straightforward: increasing the number of assets that the strategy is applied on will lead to multiplication of model's errors. If the forecasting power of the model variates, the more assets is being applied the higher errors will be. On Figure 3 there is a breakdown of total active risk into the two components – tracking error and strategy risk multiplier.


**Figure 3. Breakdown of total active risk into Tracking error and Strategy risk**

Tracking error component of active risk cannot be fully diversified as in presented in Figure 2 due to the correlation between alphas. Generally, correlation is not impacted by the number of assets. We can assume that correlation between assets is not statistically dependent on the numbers of assets. Thus, by increasing number of assists the tracking error risk $\sigma^2$ always will be leveled somewhere above 0 after the initial diversification benefit is gone. Oppositely, strategy risk will be rising with the increase of assets due to multiplication effect of applying the strategy to larger number of stocks.

## Data description and simulation method

Empirical procedures are performed on the Taiwan Stock market. After imposing liquidity and size criteria, only 489 stocks are included. Research period chosen to be between January 2012 until March 2021 giving total of 109 monthly return observations. Next monthly alphas are estimated as tracking error using TAIEX index as benchmark and $\alpha_i = r_{real} - r_{CAPM}$

Simulation procedure is used to observe performance of portfolios with different Breadth sizes. Synthetic forecast is created for each stock with exact time-series correlation of 0.1 which translates to roughly 60% success ratio. This high success ratio is observed in investors with medium to high skills in forecasting alphas. Fixing the success ratio in time-series on stock level removes the effect of high/low forecasting ability and allows for success variability on portfolio level. This is important because the same variability of forecasting skill is the core of strategy risk and as it is proved, it impacts the effect of rising breadth.

Final stage is to track portfolio performance with different number of assets. This is achieved by ranking each month the forecast's sample and picking the top n number of stocks. It is a mimicry of real-world behavior of investors because managers will rebalance their portfolio with new information at a specified interval. In such a case the smallest portfolio is with 4 assets while the biggest includes all 489 stocks.

## Risk profile of portfolios with increasing size

Our empirical examination on Taiwanese stock market shows that increasing number of assets in the portfolios consisting large number of assets has only marginal effect on active risk. Diversification benefit from larger portfolios is a non-linear function. It is interesting that initially there is a significant reduction in active risk, but after portfolio size reaches 40-50 assets then we have only minimal effects as it is presented in Figure 4:

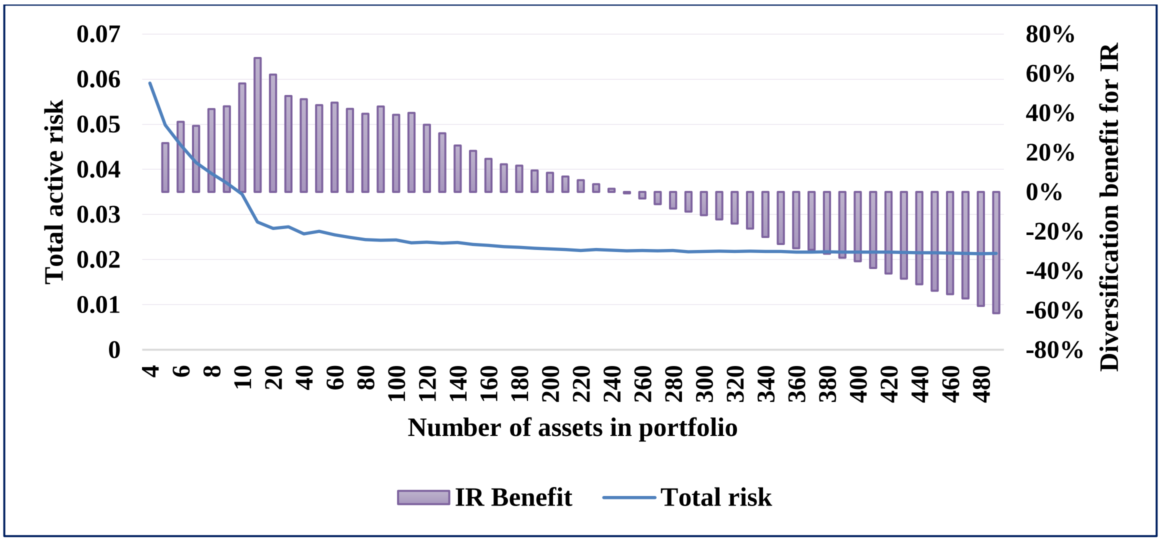


**Figure 4. Total active risk and diversification IR benefit of simulated portfolios on Taiwanese stock market**

Figure 4 provides a proof that Equation (7) correctly predicts that increasing number of assets (bets) in the active portfolio will not decrease active risk to zero. This result is not surprising or new in portfolio theory, since diversification is well studied. However, it is opposite to the classical understanding of breadth in active portfolio management. Active management in its original form suggests that investors can control the breadth of their portfolio and can maximize their IR performance by increasing number of assets (or number of investment bets). The reason behind such suggestion is that if IC can be applied for bigger breadth this will allow the investors better to benefit from forecasting power of the models due to assumption for diagonal correlation matrix which allows to diversify away the active risk. The more assets in portfolio involved the higher IR for investors is. Our result in Figure 4 though contradicts with this assumption and suggests that there are a maximum number of assets to be included, and after that any further increase is not helpful. IR benefit tracks the increase/decrease of assets compared to the initial 4 asset portfolio. When assets are increased to 20-30 IR there is a positive IR benefit due to corresponding drop in total active risk. However, after that total active stops being diversified by the increase of assets and therefore IR benefit turns negative. This loss of IR occurs because while investors get limited risk diversification, there is also alpha eating to including more and more less desirable assets.

To understand why Figure 4 presents better active portfolio management it is necessary to examine both components of active risk. FLAM's notion of diversification power of breadth is based on a risk model limited by two major components assumptions. First assumption to be relaxed is the character of strategy risk. Traditional understanding of breadth is developed on the assumption that investor's skill is constant, both in time-series and cross-sectional dimensions i.e. investor predicts with same success each stock and the error is constant. Relaxing this assumption leads to the process of increasing errors when investors increase number of assets in their portfolios. Second assumption in G&K model is non-diagonal matrix of alphas. Another key assumption to be relax is the diagonal matrix of alphas. Correlation in alphas is significant in the real world, therefore, it is important to observe when we have larger portfolios. Effect of relaxing these two assumptions is explained in (7). In the Figure 5 we present the total active risk of our portfolios broken down to its two components as in (7):

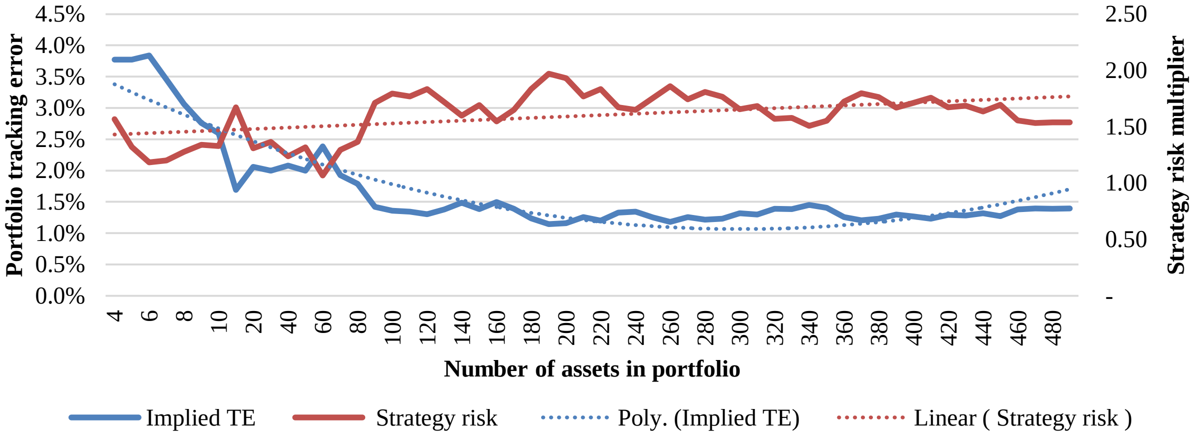


**Figure 5. Breakdown of total active risk into Tracking error and Strategy risk for simulated portfolios on Taiwanese stock market**

Our graph gives strong evidence why investors cannot fully diversify total active risk by increasing number of assets. Tracking error cannot be fully diversified since there is a significant correlation between alphas. This is a well-known process from general portfolio management where correlation prevents full diversification. It is the same in active portfolio management. Standard understanding of active management that increasing breadth will always be beneficial and investors can efficiently diversify active risk. In reality, not only those large portfolios cannot diversify risk, but they can sometimes increase it. This notion is evident from the idea of strategy risk. On Figure 5 the function of strategy risk it is also plotted if number of assets is increased. As explained in (7), it is a linear uptrend function. It is because increasing number of assets leads to increase of risk that the model will produce errors. As in equation (7) breadth is a multiplicator of strategy risk and it will increase with increasing number of assets. In some cases, increase in strategy risk can be so high that it will lead to total active risk increase. This depends on how big is $\sigma_{IC}$. If there is big volatility in forecasting skill then applying it to more assets will only increase inaccuracy of the forecast and as a result - the overall active risk in the portfolio will rise.

## Impact on IR

If we assume that active portfolio is benchmarked against broad market index like TAIEX (or S&P 500 for US), we can describe other negative side of diversification – so called alpha eating. Diversification eats alpha. Increasing number of assets in active portfolio means getting closer to the passive portfolio which diminishes alpha for active investors. The main idea of active management is to bet for "the best" assets among those in benchmark. However, when investors increase number of assets, the portfolio gets closer and closer of benchmark fading away forecasting power of the models or investors' skills (IC) to pick up the successful assets. Therefore, diversifying away total active risk comes with a cost – loss of alpha. This process is even stronger for more skilled investors with higher IC – as Figure 6 presents.

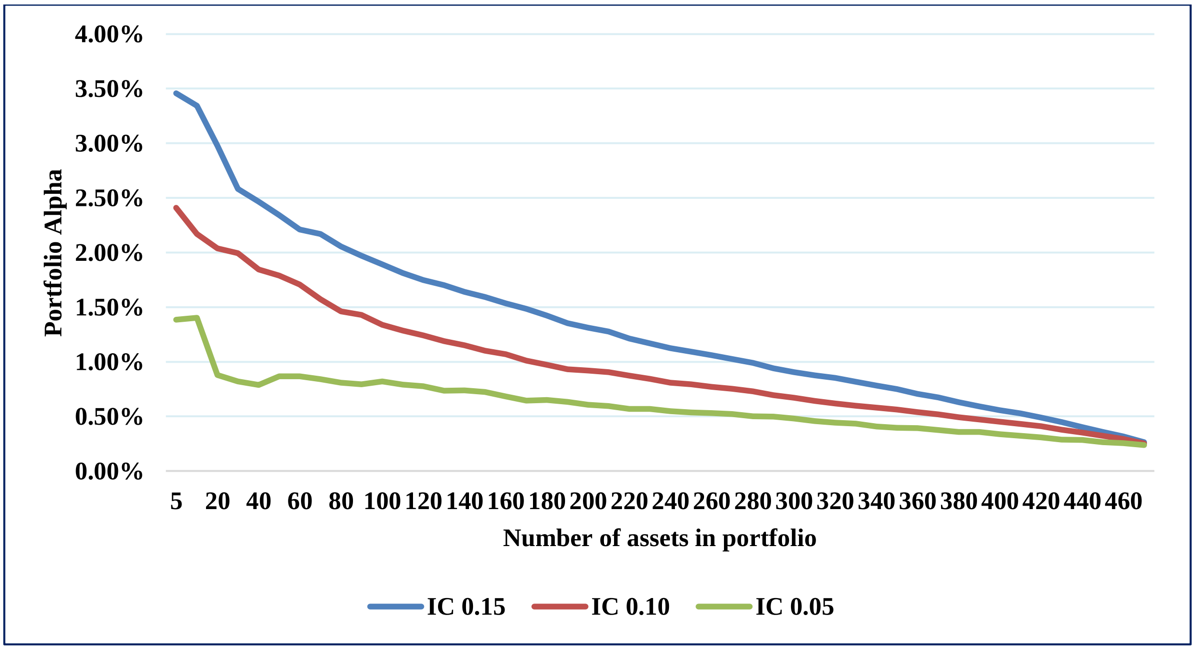


**Figure 6. Alpha eating for investors with different IC investing in simulated portfolios on Taiwanese stock market**

On Figure 6 against number of assets are plotted alphas of simulated portfolios with IC=0.05, 0.10 and 0.15 (these translate to 55%, 60% and 65% success ratio). It is evident the negative impact of increasing number of assets. Alpha eating is a paradox for active management similar to the effect of diversification in standard investing. In standard portfolio management in order to optimize risk investors have to apply diversification by increasing the number of assets in portfolio which increases the costs for investors – transaction costs. In active portfolio management investors have to sacrifice alpha to reduce active risk, i.e. their "cost" of diversification is less alpha. postulates that if investors have some IC, i.e. some ability to forecast, that investors can increase portfolio's alpha by increasing the breadth or number of assets in the portfolio. Following this rule though they must involve more and more less attractive stocks. This process makes the portfolios closer and closer to the benchmark. As a result, the portfolio is being converted from active to passive – instead of consisting of only a few "best" stocks from the benchmark, the portfolio involves additional number of stocks from the benchmark and eventually resembles the benchmark. When the portfolio fully replicates the benchmark alpha totally disappears, i.e., $\alpha_p = 0$.

Because of the process of alpha eating, it is necessary to observe diversification impact of breadth on the IR. As we know, maximizing IR should be the goal of active investors since it gives information on alpha per unit of active risk. IR of the simulated portfolio with different number of assets is plotted in Figure 7.

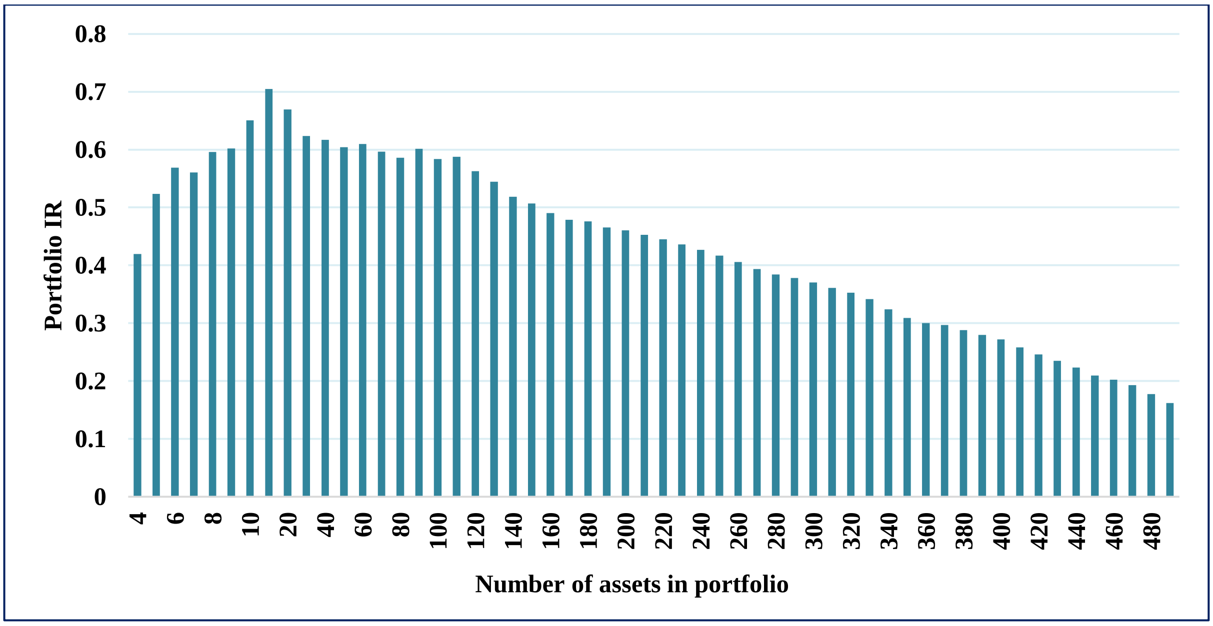


**Figure 7: Changes in IR against number of stocks for investor with 0.1 IC**

In portfolio with less than 100 assets IR follows the inverted function of total active risk. This means that for small portfolios like the ones created by individual investors, increasing number of assets to reasonable quantity will be sufficiently beneficial. In the chart we can see that active investor with medium skill (as simulated) can almost double IR if only increase number of holdings from 4 to 15.

However, in very large portfolios the drop in IR is much steeper. This is because those large portfolios are very close to passive portfolios and alpha is much diluted.

## Conclusion

Role of breadth in active portfolio management is not so straightforward as described previously by the academic theory. Investors increase their breadth by including more assets in the active portfolio. If performance is measured by IR then increased breadth should better diversify active risk and therefore increase value. However, there are two major challenges in this process. First, alpha returns are actually correlated, and this impedes diversifying tracking error. Second, increasing number of assets can increases strategy risk, because applying uncertain forecasts to more assets lead to more risk. Empirical examination on Taiwanese stock market proves that due to diversifiable strategy risk, actual IR benefit from increasing breadth is lower. Additionally, the empirical examination confirms that impact of breadth on IR is not linear. Breadth increasing can be effective for IR to around 15-25 assets (or 5%) of the total number of assets in Taiwanese stock market. After this point correlation prevents further diversification and at the same time higher strategy risk also impacts negatively IR.

## Works Citation

Ding, Z., & R. D. Martin. (2017). The fundamental law of active management: Redux.*Journal of Empirical Finance*, 43/3, 91-114.https://doi.org/10.1016/j.jempfin.2017.05.005

Gorman, L., S Sapra & R. Weigand. (2010). The Cross-Sectional Dispersion of Stock Returns, Alpha and the Information Ratio. *Investment Management and Innovation*, 7/3, 54-64.

Buckle, D. (2004). How to calculate breadth: An evolution of the fundamental law of active portfolio management. *Journal of Asset Management,* 4/6, 393–405.https://doi.org/10.1057/palgrave.jam.2240118

Ding, Z., D. Martin & C. Yang. (2020). Portfolio Turnover when IC is Time Varying. *Journal of Asset Management*, 21, 609–622. https://doi.org/10.1057/s41260-019-00145-1

Sneddon, L. (2020). Strategy design and the fallacies of breadth, *Journal of Asset Management,* 21/7, 626-635. https://doi.org/10.1057/s41260-020-00193-y

Grinold, R. & Kahn, R. (2000). *Active Portfolio Management*, 2nd edition. New York: McGraw-Hill.

Grinold, R. & Kahn, R. (1992). Information Analysis. *The Journal of Portfolio Management*, 18/3, 14-21.https://doi.org/10.3905/jpm.1992.409408

Grinold, R. (1989). The Fundamental Law of Active Management. *The Journal of Portfolio Management*, 15/3, 30-37.https://doi.org/10.3905/jpm.1989.409211

Qian, E. & Hua, R. (2004). Active Risk and Information Ratio. *The Journal of Investment Management*, 2/3, 20-34.https://doi.org/10.1142/9789812700865_0007

Ye, J. (2008). How Variation in Signal Quality Affects Performance. *Financial Analysts Journal*, 64/4, 48-61.https://doi.org/10.2469/faj.v64.n4.5

Sneddon, L. (2020). Strategy design and the fallacies of breadth. *Journal of Asset Management*, 21/7, 626-635.https://doi.org/10.1057/s41260-020-00193-y

Heinrich, L., A.Shivarova·& M. Zurek. (2021). Factor investing: alpha concentration versus diversification**.** *Journal of Asset Management*, online edition. https://doi.org/10.1057/s41260-021-00226-0

## Biographical Sketch

Plamen Patev, Ph.D. is associate professor in International College at I Shou University. He graduated from Tsenov Academy of Economics and has 30 years teaching experience and many universities in Europe. He was portfolio manager of several investment companies based in Bulgaria. Currently he is involved in research activities with ABIR Analytics. His research areas are portfolio management, active investment, emerging markets.

Kaloyan Petkov, Ph.D. is a researcher in ABIR Analytics. He has Ph.D. in finance from Tsenov Academy of Economics. Currently he is working as a Business Intelligence Analyst and it is part of ABIR Analytic research team. He has experience working with statistical software like R and BI tools like Tableau. His research interests include active investment, equity research